Problems assigned:
Undergraduates: Section 13: 5; Sections 14-17: 1(b, c), 2(a, b) 3b, 5, 10 (b, c), 11 (a, b)
Graduates: Section 13: 5, 8; Sections 14-17: 3, 5, 7, 10, 11.

Section 13:

5. Done in class.

Sections 14-17:

1. Use definition (2) of limit to prove

b. \( \lim_{z \to z_0} \overline{z} = \overline{z_0} \)

Let \( z = x + iy \) and \( z_0 = x_0 + iy_0 \). Let \( \varepsilon > 0 \).

\[
|f(z) - z_0| = |z - \overline{z_0}| = |z - z_0| = |z - z_0|
\]

So, in this case we can make \( \delta = \varepsilon \) so that if \( 0 < |z - z_0| < \delta \), \( |f(z) - z_0| < \varepsilon \)

c. Done in class.

2. Let \( a, b \) and \( c \) denote complex constants. Use definition (2) Sec 14 of limit to show that

(a) Done in class

(b) \( \lim_{z \to z_0} (z^2 + c) = z_0^2 + c \)

Let \( \varepsilon > 0 \). Then

\[
|f(z) - (z_0^2 + c)| = |z^2 + c - (z_0^2 + c)| = |z^2 - z_0^2|
\]

\[
= |z - z_0||z + z_0|
\]

\[
\leq (|z| + |z_0|)|z - z_0|
\]

Now, \( |z| = |z - z_0 + z_0| \leq |z - z_0| + |z_0| \),

So,

\[
|f(z) - (z_0^2 + c)| \leq (|z - z_0| + 2|z_0|)|z - z_0|
\]

\[
= |z - z_0|^2 + 2|z_0||z - z_0|
\]

Now we can choose \( \delta = \min \left\{ \frac{\sqrt{\varepsilon}}{\sqrt{2}}, \frac{\varepsilon}{4|z_0|} \right\} \). Then, if \( 0 < |z - z_0| < \delta \),

\[
|f(z) - (z_0^2 + c)| \leq |z - z_0|^2 + 2|z_0||z - z_0|
\]

\[
< \delta^2 + 2z_0\delta \leq \left( \frac{\varepsilon}{\sqrt{2}} \right)^2 + \frac{2|z_0|\varepsilon}{4|z_0|}
\]

\[
= \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon
\]

Which is what we wanted to show. So, \( \lim_{z \to z_0} (z^2 + c) = z_0^2 + c \)
3. Either straightforward or done in class

5. Done in class.

8. Done in class

10. Straightforward using Theorem 16 on page 49

11. Most people did this one right. Just be careful with possible zero denominators.