Math 191: Worksheet on Rates of Change

1. The volume of a sphere is $V = \frac{4}{3}\pi r^3$, where $r$ is the length of the radius.

   a) Imagine blowing up a spherical balloon. Find a formula for the (instantaneous) ROC of the volume of the balloon with respect to its radius.

   b) Find a formula for the (instantaneous) rate of change of the volume with respect to the diameter. (Remember that the diameter is twice the radius, so you’ll need to find a formula for $V$ in terms of $D = 2r$.)

   c) Find the ROC of the radius with respect to the volume. (Hint: first find a formula for $r$ in terms of $V$.) If you blow up the balloon using equal size breaths, will the first or the last breath cause a greater change in $r$?

2. If the position of an object at time $t$ is given by $s(t) = \frac{3}{2}t^2 - \frac{1}{4}t^4$, at what time $t$ in $[0, \sqrt{3}]$ is the object traveling fastest? (Hint: you’re looking for the point where $s'(t)$ is as large as possible. After $t = \sqrt{3}$, the object starts moving back towards 0.)