Every graph is assumed to be simple, unless otherwise stated.

1. Find $\omega(K_4)$ and $\omega(K_5)$. Prove your answer.

2. Prove that the least number of vertices in a cubic graph with a bridge is 10.

3. Let $G$ be a block with $\delta \geq 3$. Prove that there exists a vertex $v$ such that $G - v$ is also a block.

4. Prove or disprove: The number of cliques of a graph $G$ does not exceed $\omega(G)$.

5. Characterize the adjacency matrix of a bipartite graph.

6. Let $G$ be a connected graph with adjacency matrix $A$. What can be said about $A$ if:
   
   (a) $v_i$ is a cutvertex?
   (b) $v_i, v_j$ is a bridge?

**Extra Problems for Graduate Students:**

7. Let $b(v)$ be the number of blocks to which a vertex $v$ belongs in a connected graph $G$. Then the number of blocks of $G$ is given by

   $$b(G) - 1 = \sum_{v \in V(G)} (b(v) - 1),$$

   where $b(G)$ is the number of blocks of $G$. 