1. Prove that a graph and its complement can not both be disconnected.

2. In a connected graph any two longest paths have a vertex in common.

3. A tree has diameter 2 if and only if it is a star.

4. Prove that every self-complementary graph either has $4n$ or $4n+1$ vertices.

5. Prove that if a tree has $n$ vertices then it has $n - 1$ edges.

6. Prove that every tree is a bipartite graph.

Extra Problems for Graduate Students:

7. Prove that a closed walk of odd length contains a cycle.

8. Every graph contains at least two vertices of equal degree.