1. Draw all the graphs with 5 vertices. (Hint: There are 34 different ones).

2. Draw graphs \( G_1, G_2, G_3, \) and \( G_4, \) each with five vertices and eight edges satisfying the following conditions:
   
   (a) \( G_1 \) is a simple graph.
   
   (b) \( G_2 \) is a non-simple graph containing no loops.
   
   (c) \( G_3 \) is a non-simple graph containing no multiple edges.
   
   (d) \( G_4 \) is a non-simple graph containing both loops and multiple edges.

3. Let \( G \) be the following labeled graph:

Which of the following graphs are subgraphs of \( G \)?

4. Let \( G \) be a graph with four vertices and degree sequence \((1, 2, 3, 4)\). Write down the number of edges of \( G \) and construct such a graph.

5. Prove the 4 corollaries for the handshaking lemma.

6. Prove that the following pairs of labeled graphs are isomorphic.
7. By suitably labeling the vertices, show that the following unlabeled graphs are isomorphic.

Extra Problems for Graduate Students:

8. Prove that there is no graph with seven vertices that is regular of degree 3.

9. Prove that if $G$ is a simple graph with at least two vertices, then $G$ has two or more vertices of the same degree.