1. Find all normal subgroups of the dihedral group $D_{12}$. Prove your claims.

2. Let $G$ be a group of order $p^\alpha$ for some $\alpha \geq 0$ and $p$ a prime number. Show that $G$ has a normal group of order $n$ for every divisor $n$ of the order of the group.

3. Let $G$ be a finite group acting transitively on a finite set $S$ with $|S| \geq 2$. Prove that there exists an element $g \in G$ which does not have any fixed points, i.e. $g \cdot s \neq s$ for all $s \in S$.

4. Let $G$ be a finite group.
   (i) Prove that elements in the same conjugacy class have conjugate centralizers.
   (ii) If $c_1, \ldots, c_r$ are the orders of the centralizers of elements from the distinct conjugacy classes prove that
        \[
        \frac{1}{c_1} + \ldots + \frac{1}{c_r} = 1.
        \]

5. Let $H$ be a proper subgroup of a finite group $G$. Show that $G$ is not the union of all the conjugates of $H$. 