1. Let $G$ be a finite group, let $H$ be a subgroup of $G$ and let $N \leq G$. Prove that if $|H|$ and $|G : N|$ are relatively prime then $H \leq N$.

2. Prove that if $N$ is a normal subgroup of a finite group $G$ and $(|N|, |G : N|) = 1$ then $N$ is the unique subgroup of $G$ of order $|N|$.

3. Let $p$ be a prime and let $G$ be a group of order $p^a m$, where $p$ does not divide $m$. Assume $P$ is a subgroup of $G$ of order $p^a$ and $N$ is a normal subgroup of $G$ of order $p^b n$, where $p$ does not divide $n$. Prove that $|P \cap N| = p^b$ and $|PN / N| = p^{a-b}$. [The subgroup $P$ of $G$ is called a Sylow $p$-subgroup of $G$.]

4. Prove that $S_n = < (12), (12 \ldots n) >$.

5. Prove that $A_n$ contains a subgroup isomorphic to $S_{n-2}$ for all $n \geq 3$. 