1. (10 pts) Consider the following partially decoupled system:

\[
\frac{dx}{dt} = 3x
\]
\[
\frac{dy}{dt} = x - y.
\]

Find the general solution for this system.

\[
\frac{dx}{dt} = 3x \implies x(t) = k_1 e^{\int 3 \, dt} = k_1 e^{3t}
\]
\[
\frac{dy}{dt} = x - y \implies \frac{dy}{dt} + y = k_1 e^{3t}
\]
\[
\mu(t) = e^{\int dt} = e^t
\]
\[
\implies y(t) = \frac{1}{e^t} \int e^t \cdot k_1 e^{3t} \, dt
\]
\[
= \frac{1}{e^t} \int k_1 e^{4t} \, dt
\]
\[
= e^{-t} \left( \frac{k_1 e^{4t}}{4} + k_2 \right)
\]
\[
= \frac{k_1}{4} e^{3t} + k_2 e^{-t}
\]

General Solution: \( y(t) = (x(t), y(t)) = \left( k_1 e^{3t}, \frac{k_1}{4} e^{3t} + k_2 e^{-t} \right) \)
2. (10 pts) Consider the following first order linear system of differential equations:
\[
\frac{dY}{dt} = \begin{pmatrix} 2 & 1 \\ 0 & -3 \end{pmatrix} Y.
\]

(a) Let \( Y_1 = (e^{2t}, 0) \) and \( Y_2 = (e^{-3t}, -5e^{-3t}) \). Show that \( Y_1 \) and \( Y_2 \) are both solutions to the system.
\[
\frac{dY_1}{dt} = (2e^{2t}, 0) \quad \text{Let } A = \begin{pmatrix} 2 & 1 \\ 0 & -3 \end{pmatrix}.
\]
\[
\frac{dY_2}{dt} = (-3e^{-3t}, 15e^{-3t})
\]
\[
AY_1 = \begin{pmatrix} 2 & 1 \\ 0 & -3 \end{pmatrix} (e^{2t}, 0) = (2e^{2t}, 0) = \frac{dY_1}{dt} \Rightarrow Y_1(t) \text{ is a sol.}
\]
\[
AY_2 = \begin{pmatrix} 2 & 1 \\ 0 & -3 \end{pmatrix} (-5e^{-3t}) = \begin{pmatrix} 2e^{-3t} - 5e^{-3t} \\ 15e^{-3t} \end{pmatrix} = (-3e^{-3t}) \frac{dY_2}{dt} \Rightarrow Y_2(t) \text{ is a sol.}
\]

(b) Are these solutions linearly independent? Show your work and explain your answer.

\[Y_1(0) = (1, 0) \quad Y_2(0) = (1, -5)\]

Since \( Y_1(0), Y_2(0) \) are not multiples of each other, then \( Y_1(0), Y_2(0) \) are linearly independent.

Hence \( Y_1(t), Y_2(t) \) are linearly independent.