Name: Solutions

Banner ID: 

Note: Make sure you show sufficient work for your solutions. Correct answers without sufficient work will receive no credit.

1. (14 pts) Compute the following limits if the exist and if they don't exist, explain why.

   (a) (5 pts) \( \lim_{x \to 10} \frac{\sqrt{x-6}-2}{x-10} \)

   \[
   = \lim_{x \to 10} \frac{\sqrt{x-6} - 2}{x - 10} \cdot \frac{\sqrt{x-6} + 2}{\sqrt{x-6} + 2} \cdot \frac{1}{x-10} = \lim_{x \to 10} \frac{x - 6 - 4}{(x-10)(\sqrt{x-6} + 2)}
   \]

   
   \[
   = \lim_{x \to 10} \frac{x - 10}{(x-10)(\sqrt{x-6} + 2)} = \lim_{x \to 10} \frac{1}{\sqrt{x-6} + 2} = \frac{1}{2 + 2} = \frac{1}{4}
   \]

   (b) (5 pts) \( \lim_{h \to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h} \)

   \[
   = \lim_{h \to 0} \frac{\frac{3 - (3+h)}{3(3+h)}}{h} = \lim_{h \to 0} \frac{-h}{3(3+h)} = \lim_{h \to 0} \frac{-h}{3(3+h)} \cdot \frac{1}{h}
   \]

   
   \[
   = \lim_{h \to 0} \frac{1}{3(3+h)} = -\frac{1}{9}
   \]
(c) \( \lim_{x \to 0} \sin \left( \frac{1}{x} \right) \) dne b/c the function \( f(x) = \sin \frac{1}{x} \) oscillates infinitely many times from -1 to 1 near \( x=0 \)

2. (6 pts) Find the points of discontinuity for the function \( f(x) \) below and classify them (i.e., removable, jump or infinite discontinuities). Also, explain why these are all the points of discontinuity.

\[
f(x) = \begin{cases} 
  x^2 + 3 & x < 1 \\
  10 - x & 1 \leq x \leq 2 \\
  6x - x^2 & x > 2
\end{cases}
\]

For \( x < 1 \) \( f(x) \) is a polynomial and hence continuous

For \( 1 < x < 2 \) \( f(x) \) is a polynomial and hence continuous

For \( x > 2 \) \( f(x) \) is a polynomial and hence continuous

So the only points of possible discontinuities are \( x = 1 \) & \( x = 2 \)

\[
\begin{align*}
  x = 1: & \quad \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 + 3 = 1 + 3 = 4 \\
  & \quad \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 10 - x = 10 - 1 = 9 \quad \Rightarrow \lim_{x \to 1} f(x) \text{ dne} \quad \& \text{At } x = 1 \text{ } f(x) \text{ has a jump discontinuity.}
\end{align*}
\]

\[
\begin{align*}
  x = 2: & \quad \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 10 - x = 10 - 2 = 8 \\
  & \quad \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 6x - x^2 = 12 - 4 = 8 \\
  & \quad f(2) = 10 - 2 = 8 \quad \Rightarrow \lim_{x \to 2} f(x) = 8 = f(2) \quad \Rightarrow f \text{ is continuous at } x = 2.
\end{align*}
\]

So \( f(x) \) is continuous everywhere but \( x = 1 \), where it has a jump discontinuity.