Preface

In introducing his essays on the study and understanding of nature and evolution, biologist Stephen J. Gould writes:

> [W]e acquire a surprising source of rich and apparently limitless novelty from the primary documents of great thinkers throughout our history. But why should any nuggets, or even flakes, be left for intellectual miners in such terrain? Hasn't the *Origin of Species* been read untold millions of times? Hasn't every paragraph been subjected to overt scholarly scrutiny and exegesis?
>
> Let me share a secret rooted in general human foibles. . . . Very few people, including authors willing to commit to paper, ever really read primary sources—certainly not in necessary depth and completion, and often not at all. . . .
>
> I can attest that all major documents of science remain chock-full of distinctive and illuminating novelty, if only people will study them—in full and in the original editions. Why would anyone *not* yearn to read these works; not hunger for the opportunity [99, p. 6f]?

It is in the spirit of Gould's insights on an approach to science based on primary texts that we offer the present book of annotated mathematical sources, from which our undergraduate students have been learning for more than a decade. Although teaching and learning with primary historical sources requires a commitment of study, the investment yields the rewards of a deeper understanding of the subject, an appreciation of its details, and a glimpse into the direction research has taken.

Our students read sequences of primary sources. These provide authentic motivation for seminal problems, and trace the creation of new concepts and techniques for their solution through the centuries. The broader mathematical and social context provided by primary historical sources allows technical elements to appear in their proper place, understood and appreciated as by the creators themselves. Students will even find themselves asking many of the same questions the pioneers did, and answering these for themselves within the historical path of human discovery, thereby engendering a sense of adventure and immediacy, along with deeper motivation and a real grasp of the scope of each subject.

Primary sources also inject students directly into the process of mathematical research. They become active participants at the cutting edge of their own knowledge, experiencing actual research through grappling with the writings of great thinkers of the past. This creative immersion into the challenges of the past helps students better understand the problems of today. Finally, students gain a more profound technical comprehension, since complexity is introduced gradually and naturally.

Here we present four independent chapters, each a story anchored around a sequence of selected primary sources showcasing a masterpiece of mathematical achievement. Our stories in brief are these:

1. The dynamic interplay between the discrete and continuous in mathematics stretches from Zeno's paradoxes and Pythagorean geometric number theory to the present, aiming to quantify exactly how separated, distinct, and finite objects blend with connected, homogeneous, and infinite spaces. Today the bridge between the continuous and discrete is more important than ever, with digital technology increasingly emulating continuous phenomena.
2. A similarly ancient history underlies the development of algorithms for finding numerical solutions of equations. This evolution has gone hand in hand with multiple expansions of our notion of number itself, and today questions of algorithmic robustness and rates of convergence are vital for modern science, exemplified in the appearance of fractal phenomena.
3. In contrast, our contemporary understanding of curvature began more recently, relying on the emerging calculus of the seventeenth century. Impetus for comprehending curvature has ranged from attempts to develop accurate maps and clocks for navigating the world to our present efforts to understand the geometric nature and dimensionality, large and small, of the physical universe we live in.
4. Finally, number theory has been driven over several centuries by the mysterious yet crucial nature of prime numbers. Their behavior and patterns remain ever enticing and mysterious, yet they obey a few beautiful fundamental laws. Recently, prime numbers have emerged into a broader limelight, their elusive properties increasingly important to the security of modern electronic communication.

Our goal is to tell these stories by guiding readers through the words of the masters themselves.

The present work is similar in format to our earlier book *Mathematical Expeditions* [150], which chronicled the development of five mathematical topics at the beginning undergraduate level. However, the current endeavor encompasses different topics and at a higher level, and is for advanced undergraduates who know at least a year of calculus and have some maturity with mathematics at the upper division level. The book has emerged from a course at New Mexico State University taken by juniors and seniors majoring in mathematics, secondary education, engineering, and the sciences. While our focus is on the mathematics itself through the words of the masters, the richly historical nature of the presentation has encouraged professors at some colleges to use these materials for teaching the history of mathematics as well.

The book is quite flexible. The chapters are entirely independent of each other, except for minor biographical cross-referencing, so they can be read in any combination and order, or used individually to supplement another course. Moreover, the introduction to each chapter is an extensive freestanding summary of the relevant mathematics and its history. Within the chapter introduction, the reader is referred to the subsequent sections of annotated original sources. The individual sections can be read independently as well, preferably in conjunction with the introduction. In our own one-semester course, we usually focus on just one or two chapters; there is plenty of material in the book for at least two semesters. In the classroom we often work through the introduction together with students, jumping to the later sections as the sources are mentioned, asking students to read and write their own reactions and questions in advance of classroom exegesis of the primary source. The annotation after each source is there to help with sticky points, but is used sparingly in class. We have included many exercises throughout based on the original sources, and we provide extensive references for further reading, as well as some internet resources [144].

During the past fifteen years, discussion and use of history in teaching mathematics has expanded significantly, including the approach we take based on primary sources [30, 52, 71, 122, 132, 145, 146, 147, 160, 187, 215, 233]. And there are now increasingly many resource materials available to support the use of history [40, 53, 144, 150, 234]. Our own approach is to have students read primary sources directly, keeping the original notation as much as possible, translating only the words into English. We strongly encourage the reader to go beyond this book to explore the rich and rewarding world of primary sources. There are substantial collections of original sources available in English, which we have endeavored to compile in a web bibliography for using history in teaching mathematics [144]. Collected works of mathematicians are also a great resource [196].

This book has been ten years in the making, and we are grateful for the help of many people and institutions. Directors Tom Hoeksema and Bill Eamon of our university's Honors College provided extensive support and encouragement for the course from which this book grew. Our department heads Carol Walker and Doug Kurtz believed enough in this approach to help us make it a permanent part of our university curriculum. A grant from the Division of Undergraduate Education at the National Science Foundation provided extensive resources, including assistance and apprentice teaching by graduate student Karen Schlauch. Our outside NSF advisory consultants, John McCleary and Victor Katz, generously provided expert and extremely helpful advice, including diligent reading and editorial suggestions on several drafts. We also owe great thanks to the help of our libraries, particularly interlibrary loan.

Others have also provided invaluable special assistance and encouragement. Our colleague Mai Gehrke has taught the course with drafts of two chapters, and we are most grateful for her helpful suggestions. Harold (Ed) Edwards read and gave extremely valuable suggestions for our "bridge" chapter, as did Manfred Kolster and Jens Funke for the chapter on primes. We received assistance with French translation from Mai Gehrke, and with Latin from Danny Otero, Joe Ball, Jens Funke, and Marty Flashman, to whom we are very grateful. Keith Dennis always tells us how to find things, from sources to portraits, and we appreciate Andrea Bréard's help with Chinese sources. The special and generous technical assistance with file recovery offered by Ron Logan in a time of crisis went way beyond the call of duty. We also offer great thanks to Sterling Trantham for superb photography.

John Fauvel's tremendous enthusiasm, encouragement, and generous detailed suggestions over the years will never be forgotten. We are sad he is no longer alive to continue to hold us to the highest standards; we must aspire to them on our own, and can only hope that the final form of this book would meet with his approval.

The greatest credit for this book must go to our students. Without them, it would surely never have been written. We have used many versions of the manuscript with students at New Mexico State University, as well as at Vanderbilt University, and Hélène Barcelo has taught with some of our materials at Arizona State University. Our students' enthusiasm and accomplishments have convinced us that teaching with primary sources is invaluable to them, and their feedback greatly improved the book.

We are ever grateful to Ina Lindemann, from Springer, who showed great interest in our project, supported us with just the right mixture of patience and prodding, and whose enthusiasm provided much encouragement. And we thank David Kramer for very thoughtful copyediting, and Mark Spencer for his interest and shepherding through final production of the book.

The first author appreciates that his wife, Patricia, provided a peaceful and productive setting in which to write, and for relaxation, planned lively backpacking trips to England and the canyons of southern Utah. The second author thanks his wife, Maria Elena, for her unwavering love and support while this work was done, even though it led to many canceled weekend motorcycle rides together. The third author would like to thank the National Science Foundation for its generous support from both the Division of Undergraduate Education and the Division of Mathematical Sciences, permitting a fruitful excursion into differential geometry that united the author's research and teaching. The fourth author thanks his wife, Pat Penfield, for her enduring love, encouragement, and support for this endeavor; excellent ideas and incisive editorial critiques of several chapter drafts; and for showing us Stephen Jay Gould's essay quoted above. And he remembers his parents, Daphne and Ted, for their constant love, support, and inspirational role models for integrating history with science.

Las Cruces, New Mexico
April, 2006

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