In introducing his essays on the study and understanding of nature and evolution, biologist Stephen J. Gould writes:

[We] acquire a surprising source of rich and apparently limitless novelty from the primary documents of great thinkers throughout our history. But why should any nuggets, or even flakes, be left for intellectual miners in such terrain? Hasn’t the *Origin of Species* been read untold millions of times? Hasn’t every paragraph been subjected to overt scholarly scrutiny and exegesis?

Let me share a secret rooted in general human foibles. . . . Very few people, including authors willing to commit to paper, ever really read primary sources—certainly not in necessary depth and completion, and often not at all. . . .

I can attest that all major documents of science remain chock-full of distinctive and illuminating novelty, if only people will study them—in full and in the original editions. Why would anyone *not* yearn to read these works; not hunger for the opportunity [99, p. 6f]?

It is in the spirit of Gould’s insights on an approach to science based on primary texts that we offer the present book of annotated mathematical sources, from which our undergraduate students have been learning for more than a decade. Although teaching and learning with primary historical sources requires a commitment of study, the investment yields the rewards of a deeper understanding of the subject, an appreciation of its details, and a glimpse into the direction research has taken.

Our students read sequences of primary sources. These provide authentic motivation for seminal problems, and trace the creation of new concepts and techniques for their solution through the centuries. The broader mathematical and social context provided by primary historical sources allows technical elements to appear in their proper place, understood and appreciated as by the creators themselves. Students will even find themselves asking many of the
same questions the pioneers did, and answering these for themselves within the historical path of human discovery, thereby engendering a sense of adventure and immediacy, along with deeper motivation and a real grasp of the scope of each subject.

Primary sources also inject students directly into the process of mathematical research. They become active participants at the cutting edge of their own knowledge, experiencing actual research through grappling with the writings of great thinkers of the past. This creative immersion into the challenges of the past helps students better understand the problems of today. Finally, students gain a more profound technical comprehension, since complexity is introduced gradually and naturally.

Here we present four independent chapters, each a story anchored around a sequence of selected primary sources showcasing a masterpiece of mathematical achievement. Our stories in brief are these:

1. The dynamic interplay between the discrete and continuous in mathematics stretches from Zeno’s paradoxes and Pythagorean geometric number theory to the present, aiming to quantify exactly how separated, distinct, and finite objects blend with connected, homogeneous, and infinite spaces. Today the bridge between the continuous and discrete is more important than ever, with digital technology increasingly emulating continuous phenomena.

2. A similarly ancient history underlies the development of algorithms for finding numerical solutions of equations. This evolution has gone hand in hand with multiple expansions of our notion of number itself, and today questions of algorithmic robustness and rates of convergence are vital for modern science, exemplified in the appearance of fractal phenomena.

3. In contrast, our contemporary understanding of curvature began more recently, relying on the emerging calculus of the seventeenth century. Impetus for comprehending curvature has ranged from attempts to develop accurate maps and clocks for navigating the world to our present efforts to understand the geometric nature and dimensionality, large and small, of the physical universe we live in.

4. Finally, number theory has been driven over several centuries by the mysterious yet crucial nature of prime numbers. Their behavior and patterns remain ever enticing and mysterious, yet they obey a few beautiful fundamental laws. Recently, prime numbers have emerged into a broader limelight, their elusive properties increasingly important to the security of modern electronic communication.

Our goal is to tell these stories by guiding readers through the words of the masters themselves.

The present work is similar in format to our earlier book Mathematical Expeditions [150], which chronicled the development of five mathematical topics at the beginning undergraduate level. However, the current endeavor
encompasses different topics and at a higher level, and is for advanced under-
gergraduates who know at least a year of calculus and have some maturity
with mathematics at the upper division level. The book has emerged from a
course at New Mexico State University taken by juniors and seniors majoring
in mathematics, secondary education, engineering, and the sciences. While
our focus is on the mathematics itself through the words of the masters, the
richly historical nature of the presentation has encouraged professors at some
colleges to use these materials for teaching the history of mathematics as well.

The book is quite flexible. The chapters are entirely independent of each
other, except for minor biographical cross-referencing, so they can be read
in any combination and order, or used individually to supplement another
course. Moreover, the introduction to each chapter is an extensive freestand-
ing summary of the relevant mathematics and its history. Within the chap-
ter introduction, the reader is referred to the subsequent sections of anno-
tated original sources. The individual sections can be read independently as
well, preferably in conjunction with the introduction. In our own one-semester
course, we usually focus on just one or two chapters; there is plenty of material
in the book for at least two semesters. In the classroom we often work through
the introduction together with students, jumping to the later sections as the
sources are mentioned, asking students to read and write their own reactions
and questions in advance of classroom exegesis of the primary source. The
annotation after each source is there to help with sticky points, but is used
sparingly in class. We have included many exercises throughout based on the
original sources, and we provide extensive references for further reading, as
well as some internet resources [144].

During the past fifteen years, discussion and use of history in teaching
mathematics has expanded significantly, including the approach we take based
And there are now increasingly many resource materials available to support
the use of history [40, 53, 144, 150, 234]. Our own approach is to have stu-
dents read primary sources directly, keeping the original notation as much as
possible, translating only the words into English. We strongly encourage the
reader to go beyond this book to explore the rich and rewarding world of pri-
mary sources. There are substantial collections of original sources available in
English, which we have endeavored to compile in a web bibliography for using
history in teaching mathematics [144]. Collected works of mathematicians are
also a great resource [196].

This book has been ten years in the making, and we are grateful for the help
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