Preface

Nothing captures the excitement of discovery as authentically as a description by the discoverers themselves. What better place to read about the search for the origin of the Nile than the report of Sir Richard Burton on his amazing journey into the depths of the African continent? The overwhelming awe and beauty of the Grand Canyon comes alive when we hear John Wesley Powell tell of his thrilling ride down its unexplored rapids. The same holds true for the explorers of the unknown territories of the mathematical world. While a second-hand account of a mathematical expedition might seem more orderly than the description of the explorer, it will likely lack the excitement, immediacy, and insights of a story told by someone who was there. It is this excitement and immediacy of mathematical discovery that we want to convey. This book contains the stories of five mathematical journeys into new realms, pieced together through the writings of the explorers themselves. Some were guided by mere curiosity and the thrill of adventure, others by more practical motives. In all cases the outcome was a vast expansion of the known mathematical world and the realization that still greater vistas remain to be explored.

In the development of calculus, one great goal was to find methods for computing areas and volumes beyond the achievements of classical Greek geometry. The potential for applications was enormous, and once success was achieved, the sciences were revolutionized. Likewise, the prospect for methods to solve general algebraic equations fostered high hopes for important applications within mathematics, as well as in the sciences and engineering. In contrast, the quest for a solution to the so-called Fermat's Last Theorem in number theory seems whimsical, apparently motivated by nothing more than the desire to meet a challenge. Nonetheless, when the goal was finally reached, mathematics

had profited immeasurably from the effort. The same outcome crowned the attempt to conquer the treacherous world of infinite sets. What first seemed liked a hopeless fight against paradoxes and the tenets of theology turned into one of the greatest revolutions mathematics has ever seen. Finally, the apparently placid waters of geometry were hiding a rich world beyond the imagination of all but a handful of brave souls who dared explore the possibilities of a non-Euclidean view of geometry and the physical world. Begun out of a desire for mathematical elegance and completeness, this two-thousand-year quest led to a vast expansion of mathematics and fundamental applications to the theory of relativity. We will tell these stories as much as possible by guiding the reader through the very words of the mathematicians at the heart of these events.

This book is more about mathematics than it is about its history. Our goal is to throw light on the mathematical world we live in today, and we believe that its history is essential to understanding and appreciation. Our project began as a freshman honors course we have taught at New Mexico State University since 1989. The aim of the course is to introduce students from a wide variety of majors to the exciting world of mathematical discovery. Typically, some subsequently decide to major in mathematics. In the course we try to get across the thrill of exploring the unknown that motivates most mathematicians. Students see the mighty mountains that the community of mathematicians scales, sometimes through the joint effort of many generations. In the end a better understanding of the mathematical present is paired with the realization that mathematics is a living, breathing subject, facing new challenges every day. We hope that this book serves the same goals.

The book can be used in a variety of ways. The five chapters are completely independent of one another, as are largely the individual sections within each chapter. Necessarily, the level of difficulty within a chapter varies considerably. The chapter introduction and first sections can be appreciated and understood by someone with a good high school education in mathematics. The later sections require considerably more mathematical maturity. It is our vision that the book will be enticing both to the intellectually curious reader and to instructors and students as a course text. The introduction to each chapter summarizes the story historically and mathematically, and subsequent sections feature the original writings of major explorers in that particular story of discovery. The five introductions, together with selections from the other sections, can be used as a text for a mathematically oriented history of mathematics course. Individual chapters can be used in a serious mathematics appreciation course or as a supplement to another mathematics course. Most importantly, it is our hope that the text will encourage the creation of courses like the one from which it originated. In our one-semester course, we usually focus on

just two or three chapters. There is enough material in the book for at least two semesters.

Our initial inspiration to create courses in which students learn mathematics in its historical context was William Dunham's "Great Theorems" course [44, 45]. Unlike Dunham we insist on reading primary sources. (Our one compromise is the use of English translations.) We have discussed the feasibility and the many benefits of this approach in [103, 104, 105]. The resources [24, 156, 167, 168] as well as the newsletter [90] also contain much information on using history in teaching mathematics.

Our primary sources trace five central themes in the evolution of mathematics. Our selection criteria were the importance of the source as a milestone of progress and its accessibility without extensive prior preparation. In these choices and in our own commentary we make no claim to be comprehensive in breadth, detail, or the contributions of various individuals, groups, or cultures. Ours is not a history of mathematics, but rather an exploration of some exciting mathematics through its historical artifacts.

How do we use these materials in our own teaching? Usually, we work through the introduction together with the students and jump to the later sections as the sources are mentioned. The annotation after each source is there to help with sticky points, but is used sparingly. We have included many exercises based on the original sources, and welcome more from our readers. A most useful exercise is to rewrite a source in one's own words using modern notation, filling in all the missing details.

Finally, we integrate prose readings about mathematics into the course, many from the wonderful collection [130]. We provide students with questions about these readings, and written answers then form the basis for class discussion.

We strongly encourage the reader to go beyond this book to explore the rich and rewarding world of primary sources. There are substantial collections of original sources available in English, such as [13, 14, 58, 87, 122, 160, 166]. Collected works are, of course, also a great resource [142]. We have provided many references in the text for further reading.

This book has been in the making for almost ten years. It might never have been completed without the help of many people and institutions. The directors Tom Hoeksema and Bill Eamon of the NMSU honors program provided extensive support and encouragement for the course from which this book grew. Our department heads, Carol Walker and Doug Kurtz, believed enough in our approach to teaching to help us make it into a permanent addition to our curriculum. A grant from the Division of Undergraduate Education at the National Science Foundation provided extensive resources. The NSF advisory committee, consisting of Judy Grabiner, Tom Hoeksema, and Fred Rickey, gave lots of great advice, diligent reading, and editorial suggestions on several drafts. In addition, Florence Fasanelli provided sage words of wisdom at crucial times. The grant also al-

lowed us to involve a graduate assistant, Xenia Kramer, in the project. We owe her special thanks for her extensive contributions to research and writing, and for testing earlier drafts in the classroom as an apprentice teacher. The lion's share of the credit must go to our students, however, without whom this book would never have been written. We used early versions of the manuscript in classes at NMSU, as well as Cornell University and the two-year-long NSF-sponsored Young Scholars Mathematics Workshop in the Rockies, at Colorado College. Our students' enthusiasm convinced us that teaching with original sources can work, and their feedback greatly improved the book.

Many people have helped us in locating original sources. We could always rely on the expertise of Keith Dennis, as well as his wonderful private collection. Our research on Sophie Germain would have been impossible without the help of Larry Bucciarelli, Catherine Goldstein, Helmut Rohlfing of the Niedersächsische Staatsbibliothek in Göttingen, and the Bibliothèque Nationale in Paris. Thanks are also due to Dave Bayer, Don Davis, and Anne-Michel Pajus. We received assistance with translations from Hélène Barcelo and Mai Gehrke. Bill Donahue, Danny Otero, Kim Plofker, and Frank Williams contributed their expertise in Latin.

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