A graduate course on the role of history in teaching mathematics

in *Study the Masters: The Abel-Fauvel Conference*, Kristiansand, 2002, (ed. Otto Bekken et al), National Center for Mathematics Education, University of Gothenburg, Sweden, in press.

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Dedicated to the memory of my friend and inspiration John Fauvel.

We have developed a graduate course on the role of history in teaching mathematics, emerging from our program of undergraduate teaching with original historical sources and a collaboration with high school teachers. Graduate students in the course learn about use of history worldwide, and prepare a teaching module for a setting of their choice, usually based on original historical source material. We describe the course goals, impacts, pedagogical design and content, resources, results, and challenges. Further material is available at `math.nmsu.edu/~history`.

We discuss a graduate course on the role of history in teaching mathematics, with special emphasis on original historical sources. We consider the background and aims of the course, its reach and impacts on students, what new resources are produced, and the detailed content, pedagogy, and resources applied. We will also address major issues and challenges raised for all those interested in studying, using and expanding the role of history, particularly original sources, in teaching mathematics.

Our graduate course, taught regularly since 1995 at New Mexico State University, emerged from other educational initiatives begun in the late 1980s. At that time Reinhard Laubenbacher and the author developed two undergraduate mathematics courses based entirely on student study of original historical sources, i.e., studying the masters (Laubenbacher & Pengelley, 1992, 1993, 1996, 1999a), (Laubenbacher, Pengelley, & Siddoway, 1994). Subsequently other colleagues became involved in teaching and expanding the original source material in these courses, and two books based on annotated original sources have arisen (Laubenbacher & Pengelley, 1998, 1999a); teaching with history and original sources has also expanded into some regular courses in our department's curriculum. Simultaneously the author and others developed a parallel departmental initiative introducing student research projects into our calculus curriculum, as part of the movement to reform calculus teaching in the

United States (Cohen, Gaughan, Knoebel, Kurtz, & Pengelley, 1992) (Pengelley, 1991, 2000); this expanded to a partnership with local high school teachers, introducing a student project approach into high school mathematics teaching, and creating an excellent collaborative relationship with many local teachers (Knoebel, Kurtz, & Pengelley, 1994). In 1995 R. Laubenbacher and the author built on this relationship by offering a workshop graduate course "Using Historical Sources in Teaching Mathematics" for high school teachers, intended to foster incorporation of original sources in high school courses. In particular, the participating teachers would develop original source teaching modules for their high school classrooms (Laubenbacher & Pengelley, 1999b).

To our surprise, others who heard about this workshop course didn't want to be left out: middle school teachers, undergraduate and graduate students in mathematics education, and quite a number of our mathematics graduate students came knocking on our doors. Thus we catered to a very eclectic group of students in a Spring 1995 workshop. It became clear that a regular graduate course would have great benefit and appeal for graduate students, and the now biennial course, "The Role of History in the Teaching of Mathematics", was born, serving Masters and Ph.D. students in both mathematics and education. New faculty are beginning to teach the course, and we are hoping to integrate it within a mathematics education component of our mathematics Ph.D. program that is under development. Much of the material to which we will refer below for the course, and for the other initiatives mentioned above, is available at our web site (Laubenbacher & Pengelley, 1999a).

Course goals, impacts, and resources created

The overall intent is to shed historical light on what we teach and why we teach it, thereby enriching our mathematical experience, and to consider and practice the various ways of incorporating history to improve our teaching. The course combines three primary activities towards these ends. First, students should begin to study some history of mathematics, since many have little, if any, previous exposure. Second, students should study and critique materials from around the world that discuss, argue about, illustrate and analyze the many ways of incorporating history in mathematics teaching. And third, each student should become familiar with available resources, and then select a particular historical topic and relevant materials, tailored to their own interests and circumstances, for developing a substantial teaching module; much of each graduate student's work in the course is spent creating and polishing this module. Usually it is based directly around original source material, with an intended curricular setting at any chosen level, and is written for both mathematics students and instructors. The module goes through many revision cycles with critiquing from the instructor and fellow students. Often the modules result in public presentations, classroom testing, and sometimes in publication (Goar, 1999). Some graduate students also go on to use their materials in future teaching, and some find that the course and these experiences have a substantial positive impact on their attractiveness as potential teachers or postdoctoral fel-

lows to college and university employers. Some students continue on to further graduate work in history of mathematics and its use in teaching.

All the modules that graduate students have developed are listed in (Laubenbacher & Pengelley, 1999b), and copies of some are available upon request. The titles, authors, topics, and original source authors of some of the teaching modules prepared by graduate students are given in Appendix 1.

Course pedagogy, content, and resources

The author conducts the course in a seminar style, with discussion format and regular student presentations and written and oral critiques. Numerous resource materials are used to introduce graduate students to the breadth of what is available, and to the many worldwide pedagogical facets of using history in teaching mathematics, with special emphasis on using original source material in the classroom at all levels from elementary school to graduate school. Many of the course resources are provided or referenced at our web site (Laubenbacher & Pengelley, 1999a). In particular, *Some Selected Resources for Using History in Teaching Mathematics* (Laubenbacher & Pengelley, 1999c) includes a list of published original source collections, a link to *A Bibliography of Collected Works and Correspondence of Mathematicians* (Rockey, 1991), and many other types of resources on and for using history in teaching, such as (Calinger, 1996)(Swetz, 1995). Also on our web site is a rough course syllabus for instructors and students (Pengelley, 2001).

Some examples of the nature of course activities and resource materials in the context of individual student assignments and class discussion are:

- Write about your own background in relation to the course
- Introduce students to resource materials, including histories of mathematics, source books of original sources, and bibliography of collected works
- Select a historical thread, read about it in a history of mathematics, write on it, and present and lead a class discussion
- Select and write critiques of several previous teaching modules using historical sources, developed in the course in previous years; use a prepared list of questions *Critique of modules* (Pengelley, 1998), here in Appendix 2
- Write critiques of articles *Origin and Evolution of Mathematical Theories: Implications for Mathematical Education* (de Guzmán, 1993) and *Recovering Motivation in Mathematics: Teaching with Original Sources* (Laubenbacher, Pengelley, & Siddoway, 1994), discuss in class
- Read and write an analysis on part of our first book based on annotated sources, *Mathematical Expeditions: Chronicles by the Explorers* (Laubenbacher & Pengelley, 1998), as teaching material

- Read in histories of mathematics and original sources to devise ideas for individual project modules, make individual timetables for project work and completion, and work on developing them over time. The instructor meets regularly with individual students to help with projects
- Critique the article *The ABCD of using history of mathematics in the (undergraduate) classroom* (Siu, 1997), and discuss
- Choose and critique several more previous teaching modules developed in the course
- Discuss and critique the ICMI study discussion document *The role of the history of mathematics in the teaching and learning of mathematics (1997–2000)* (Fauvel & van Maanen, 2000a), which led to the book *History in mathematics education* (Fauvel & van Maanen, 2000b)
- Compare and contrast parts of *Contextual History of Mathematics* (Calinger, 1999) and *History of Mathematics* (Katz, 1998); these books take very different approaches to history of mathematics
- Critique *The Development of Algebra: Confronting Historical and Psychological Perspectives* (Sfard, 1995), and discuss. This article delves into the nature of learning algebra in relation to its historical development, and is a demanding and provocative topic for class time and effort
- Oral project presentations and critiques in class. Test the project module in a teaching classroom, if possible
- Final semester projects due, after numerous revision cycles, reports, and class presentations. These become the final course modules, added to the archived course materials
- Make public presentations of teaching module projects in a seminar or at a conference
- Encourage preparation of selected projects for publication.

Issues and challenges

In the course, we discuss pedagogical, educational, and institutional issues surrounding the role of history, and in particular original sources, in teaching mathematics. This involves questions of importance, efficacy, and challenges of implementation. We will mention here just a few of the questions which arise.

- What is the nature of the evidence and/or reasoning that history of mathematics can play a valuable role in mathematics education, and how essential is this role? What form should the evidence take? How compelling is the evidence, and how important is it that there should be compelling evidence?

- Are useful materials available for instructors and students to incorporate history in various ways? It seems that this is improving rapidly on all fronts, thanks to the efforts of many dedicated people.
- Do teachers have the background and preparation to make it easy and effective for them to incorporate history? This is greatly complicated by the fact that most teachers at every level have learned little about the history of mathematics from their own mathematical education.
- Are relevant original sources available, and how usable are they for teachers and students? Are they available as an original or a translation in a language that can be used in instruction? Have they been annotated and placed in context for teaching? Are they easy to read, understand, and incorporate into existing curricula? What strengths of perspective and insight will a given original source bring to a particular teaching setting, and what will the challenges be in using it?
- How practically and efficaciously can history be incorporated in existing curricula? Does history "slow down" instruction? Does it make it more efficient and effective in the long run, by helping students with motivation and linkages? Does history make the mathematics in the classroom more difficult, or easier?

These are all questions that need addressing if one wishes to foster an increase in the number of teachers involved in using history and original sources in teaching mathematics.

Appendix 1: Selected modules written by graduate students

Graphs and More Graphs (Daniel Fish, NMSU) Statistics, Florence Nightingale

Restoring the Square - The Methods of Al-jabr (Mike Goar, Mayfield High School) Algebra and Geometry, al-Khowarizmi

Getting to the Root of the Problem: A Hint from René Descartes (Mike Goar, Mayfield High School) Algebra, René Descartes

Linear Programming and the Simplex Algorithm (Jeff Holland, NMSU) Linear Algebra, George Dantzig

The Bernoulli Numbers (Tuesday Johnson, NMSU) Number theory, Jacob Bernoulli

Every Integer is a Sum of Four Squares (Tuesday Johnson, NMSU) Number Theory, Leonhard Euler

Ptolemy's "Table of Straight Lines in a Circle" (Mary Joy, Mayfield High School) Trigonometry, Claudius Ptolemy

Developing the Quadratic Formula (Glendel DeVon Love, Las Cruces High School) Algebra, al-Khowarizmi

The Euclidean Algorithm (Glendel DeVon Love, Las Cruces High School) Number Theory and Geometry, Euclid

Egyptian Mathematics (Cynthia Oszman, White Sands Middle School) Arithmetic, The

Rhind Papyrus

Mathematics Using Sumerian Economic Texts (Cynthia Oszman, White Sands Middle School) Arithmetic, Texts from the Drehem archive

Fibonacci's Rabbit Problem, and Consecutive Odd Numbers (Carmen Pareo, Gadsden High School) Number Theory, Leonardo Fibonacci

Operation Tangent (Michael Quintana, NMSU) Tangent Lines to Curves, Rene Descartes and Pierre de Fermat

Diophantine Equations (Gina Rivera, Vista Middle School) Algebra and Number Theory, Diophantus

The Cayley-Hamilton Theorem (Eric York, NMSU) Linear Algebra, Arthur Cayley and William Rowan Hamilton

Cramer's Rule and the Determinant (Rebecca Brown, NMSU) Linear Algebra, Gabriel Cramer

Olivier and Abel on Series Convergence: An Episode from Early 19th Century Analysis (Michael Goar, NMSU) Calculus and Real Analysis, Niels Henrik Abel

Arthur Cayley and the Origins of Group Theory: An Introduction to Abstract Algebra (Daniele Richardson, NMSU) Abstract Algebra, Arthur Cayley

Appendix to "Restoring the Square: The Methods of Al-Jabr" (Daniele Richardson, NMSU) Algebra and Geometry, al-Khowarizmi

Using Diophantus of Alexandria to Teach Algebra (Shelly Hangen, NMSU) Algebra, Diophantus

Using Stigler's "Diet Problem" to Teach Linear Programming (Rumiya Masagutova, NMSU) Linear Programming, George Stigler

Logic Through the Looking Glass: Learning the Basics of Symbolic Logic Through the Works of Lewis Carroll (Gloria Johnson, NMSU) Symbolic Logic, Lewis Carroll

Navigation and Map Making (Karen Ondo, NMSU) Geometry, Trigonometry, and Calculus, Gerardus Mercator and Edward Wright

Napierian Logarithms: Who, When and How (Mary Williams, NMSU) Logarithms, John Napier

The Theory of Galois: A Historical Approach (Rebecca Pablo, NMSU) Galois Theory, Solutions of Equations, Evariste Galois

Appendix 2: Critique of modules

Math 561, Spring 1998
Critique of Modules

Consider these questions:

1. What is the intended audience?
   (a) Is this audience well-considered throughout the module?
   (b) Is the information appropriate?
2. Is the information given accessible to teachers? Students?

3. Is the writing clear?
4. What is the purpose of studying the material? (See also #14 and #15)
    (a) How does it fit with the curriculum? (See also #8)
5. Has the proper amount of original source material been selected?
6. Is there appropriate historical content?
    (a) Ambient social history
    (b) Biography
    (c) Mathematical content
    (d) Notation, mathematical paradigms
    (e) Balance of math vs. history
7. Is the mathematical annotation appropriate? Too little? Too much?
8. How will it be used in the classroom?
    (a) What actually happens in class?
    (b) How will assignments work? (See also #9)
9. Are there exercises, and are they appropriate?
    (a) Appropriate difficulty level?
    (b) Are they well chosen to demonstrate concepts?
10. Is everything correct?
11. Are the references adequate and useful?
12. Is the reader enticed into further exploration? (See also #13)
13. Will this motivate students?
14. What will students learn?
15. Will this help with something students have trouble with?

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