From lecture to active learning: Rewards for all, and is it really so difficult?

David Pengelley

August 29, 2017

In the centuries-honored *I-You* teaching paradigm, an instructor provides first contact with new material via lecture, then expects students to solve mathematics problems outside class: *I* lecture, then *You* do homework alone[1]. Yet an enormous body of research now points to *I-You* being an ineffective pedagogy for student learning, and indicates better alternatives [2, 3, 4, 6, 7, 9, 11, 12, 13, 14, 15, 16, 17, 19, 20, 24, 25].

Most recently, presidents of 14 mathematics professional societies joined in the statement *Active Learning in Post-Secondary Mathematics Education* [5], from which I quote briefly: "A wealth of research has provided clear evidence that active learning results in better student performance and retention than more traditional, passive forms of instruction alone. ... In recognition of this, we call on institutions of higher education, mathematics departments and the mathematics faculty, public policy-makers, and funding agencies to invest time and resources to ensure that effective active learning is incorporated into post-secondary mathematics classrooms. ... [Moreover,] active learning confers disproportionate benefits for STEM students from disadvantaged backgrounds and for female students in male-dominated fields". I add that these benefits accrue while not disfavoring high-achieving or more experienced students, or any demographic group [6, 11, 14, 15].

So if evidence is now so overwhelming that all our society presidents exhort us to replace *I-You* with something better, then with what exactly, and is that hard to do? My aim here is not to present yet another research study, nor to survey varying features of active learning teaching methods. Rather, I will provide an analysis from my multi-decadal arc of personal development, leading to specific teaching philosophy about the connections between before, during, and after the classroom, along with encouragement for readers who may worry that alternatives to lecture seem complicated and time consuming. My main message is that it doesn't have to be difficult to create active learning for your students, and that there are tremendous rewards for the instructor as well as for students. My primary goal is to entice hesitant readers to take a teaching plunge.

Let me pose some questions to guide what follows: Would it be desirable, or feasible, to expect students to prepare before class via first-contact reading, writing, and problem work, so that student classroom activities could build on this, with plenty of in-class feedback from fellow students and the instructor, followed at home by higher level work? What might be the motivation and benefits for both students and instructors? How hard would it be to

[1] The *I-You,* etc., terminology used here is discussed in [8].

shift toward this pedagogy? Won't the world fall apart if I stop lecturing? Am I trapped? Are different teaching materials required? What needs changing, and what doesn't? How much time would it take from research or other faculty responsibilities? And what about the fearsome elephant called coverage, whom many instructors believe resides in their classroom?

Different active learning paradigms have a panoply of monikers[2], but they all share the two in-classroom features supported by the research cited above—first, reduce or eliminate lecture, and second, devote substantial classroom time to student involvement in mathematical work that receives immediate feedback from other students and from the instructor. Such paradigms clearly involve more of the ingredients *You* and *We*, and considerably less of *I*. Concomitantly, students will be more in charge of and responsible for their own learning, while instructors will have increased responsibility to guide and manage student work.

But going further, I particularly wish to explore how the above in-classroom changes should be connected with what students do outside the classroom. I intend to think through the interlinked details, especially the interplay between student preparation before class and what can actually happen in the classroom, and then after class. This leads to a motto on first contact and reading, and to a particular alternative paradigm that emerged from my own decades evolving a non-lecture pedagogy in sixteen different courses[3] at all undergraduate and graduate levels. It has the two in-classroom features already espoused, but also additional linked ingredients that I have found crucial and highly beneficial.[4]

I can offer expertise only from my own example, and I applaud that others discover their own paths. But what I am especially able to offer is that my nuts and bolts have been refined in detail in many different courses, and I will share what has worked and what hasn't, and in which courses, how issues of time and coverage have worked out in practice, and student actions and reactions. Equally important, since a shift toward active learning likely seems highly unfirm ground for those instructors primarily used to lecturing, I will give details that provide some needed assurance, such as how demanding and time-consuming this is or isn't, and what the pitfalls are. I will also more generally address inertia, challenges, efficacy, fear, burnout, and rewards, not only for students, but at least as critically important and motivating, for instructors, since rewards for instructors are perhaps crucial to overcome hesitancy.

Lecture

From my early life as an *I-You* student, I remember occasional inspiration from lectures, but there was not much learning there that enabled me to complete anything but rote homework. After all, lecture usually primarily involves the instructor demonstrating that she/he can do the mathematics. But this rarely helps a student actually be able to do much

[2] E.g., inquiry based learning [11, 14], interactive engagement [6], total quality management [26], just in time teaching [10, 18], peer instruction [16], flipped or inverted classroom [1].

[3] Lower division: Calculus I,II,III, Discrete Mathematics, Discrete Mathematics for Computer Science, Mathematics Appreciation, Spirit and Evolution of Mathematics.

Upper division: Abstract Algebra, Real Analysis, Combinatorics, Topology, Geometry, Number Theory, Great Theorems.

Ph.D. level: Topology I,II.

[4] See `https://www.math.nmsu.edu/~davidp/` for assignment examples, as well as guidelines for students, grading, and logistics.

mathematics, any more than a swimming instructor demonstrating an hour of beautiful swimming techniques successfully teaches a beginner how to swim various strokes. As a student, I survived and prospered despite a lecture setting, only by reading text material, over and over, integrated with tackling homework challenges. I now realize this was essentially autodidactical, my instructor's role chiefly being to provide a schedule, expectations, written feedback on homework, and evaluation via exams.

My subsequent four decades teaching thousands of students suggests that few students will very successfully self-teach as above. The paradox for readers of this article is that we are probably the most notable group of exceptions; we are among the rare survivors or "thrivers" of the *I-You* approach. But I expect we all have frequent conversations with random adults, or even with our own university colleagues in different disciplines, all former *I-You* students of mathematics, and we constantly receive confirmation from them that the average *I-You* experience was a dramatic failure leaving many scars.

During the years I lectured, I listened countless times as students told me "I know the math. I understand perfectly when you lecture, but then I just can't do the problems at home." Of course in actuality this meant they didn't really "know the math", but I didn't know what I could do to help, other than to lead them through homework problems. In retrospect, for all but possibly inspiration or rote learning, my lecturing was ineffective, despite all my best efforts, and notwithstanding my students' encouraging lauding of my lectures, their desire for it, and belief in it. And since it wasted precious classroom time, it was inefficient as well. In fact, classroom lecture will surely become largely obsolete in the future, since with modern technology any recorded lecture can be viewed by anyone, anytime, anywhere. How long will it take university administrators to conclude that they need not employ highly-paid professors to add more lectures to the increasing number already archived? In short, professors had better have something more to offer students than yet more lectures on settled subjects. Of course we will all claim that our students really do need much more than a lecture to succeed, and that we can provide that. So isn't that what we should home in on? How do we both challenge students and guide and support their work as learners in truly productive ways?

First contact with new ideas

In rethinking the *I-You* paradigm, much revolves around the question of first contact[5]: How and when should a student first be exposed to new material? In mathematics especially, absorbing and making sense of substantial new ideas with any depth is usually a slow, highly individualized, intellectually messy business. Lecture is by nature time-limited, one-size-fits-all, and totally incompatible with the need to say "Stop, wait a minute, let me think that through." In short, lecture is on its face a poor venue for first contact with demanding new material, despite our natural inclination to the contrary, that as instructor we can help students get started digesting new ideas by offering them a lecture first.[6]

Today it is not uncommon to modify *I-You* a little, involving students in some interaction while lecturing, by asking questions or having students work on related problems during

[5] I was enormously influenced by the ideas of Barbara Walvoord [21, pp. 53–63] on first contact with new material, as beautifully described to me by Virginia (Ginger) Warfield.

[6] For more on the disconnect between the role of lecture for instructors and students, see [23].

class. Thus *I* becomes intertwined with *We*, and the paradigm morphs into *I-We-You*. However, in-class student problem work based on just-introduced first-contact material can suffer the same drawbacks as student attempts to absorb from lecture, since one generally needs substantial thinking time to digest new mathematical ideas before being able to do anything useful with them.

So if in-class lecture provides poor first contact, and even non-passive first-contact classroom activities have their drawbacks, then perhaps first contact, and maybe even first problem work, might better occur *before* class, and something entirely different can happen *during* class. This could lead to the recently named flipped or inverted classroom, in which lecture and homework switch venues, with students watching recorded lectures before class and working together on mathematics during class. But watching recorded lectures has several drawbacks too. It is in many ways at least as passive as watching a lecture live. And it suffers the same cognitive drawbacks already listed above for in-class lectures, unless a student were to frequently hit pause and replay, all the while thinking things through deeply, which is unlikely to happen without additional pedagogical structure. Finally, the amount of instructor time and effort needed to create recorded lectures is enormous, unless one utilizes someone else's. My own conclusion, arrived at over twenty years ago before recorded lectures were even practical, was to evict lectures entirely and evolve new paradigms.

How then do I want students to obtain meaningful first contact with new material before class? And what consequently can most usefully happen during class?

Reading and the lecture-textbook trap

My simplest answer for meaningful first contact is to thoughtfully engage high-quality reading (or, in some circumstances, something equivalently demanding), which begins to transform the paradigm from *I-* to *You-...* . However, while reflection and thinking stimulated by reading can be extremely powerful, simply exhorting students to read the book before class rarely works, since they seldom read as suggested, nor get much out of it even if they do read.

There is a gaping trap here, a truly vicious cycle in which students don't read beforehand when they know the instructor will lecture, and instructors lecture in large part because they know students haven't read. Breaking out of this lecture-textbook trap was the most difficult teaching problem I ever had to solve, but all else flowed from it. It was my responsibility to break the cycle by insisting (to self and students) that I will not lecture, and instead arranging for in-class activity to be built on a foundation of high-quality student preparation. And I needed my students to buy into it. A guiding motto arose: "Never lecture on something students can read instead."

Written response to reading

But, seemingly making matters even more challenging, reading alone is insufficient, even if I convince students to do it. For two reasons, I should guide students to reflect and think critically about what they read, to make connections, and to respond in writing: First, it is writing that will hone their intellectual engagement and critical thinking and analysis;

second, I really need to know their responses to reading in order to prepare for a productive non-lecture class session.

I find that for lower-division students a couple of well-crafted questions from me about the reading are essential; whereas upper-division students can be educated to write well without me providing specific questions. I expect students at all levels to write their own good questions about their reading, and to write which new concepts are confusing, what was well explained and interesting, what they had to reread but eventually understood, and what connections they see to other ideas. The questions I make about the reading are simply designed to stimulate students to read carefully and think it over, and to catalyze and help guide class discussion. And sometimes the reading questions aren't questions at all, but brief tasks based directly on the reading.

As an example, in a first calculus course, after reading a textbook section introducing the derivative, my reading questions might be: "Explain in your own words what your understanding is of the idea of the derivative of a function." and "What are the different mathematical and physical interpretations we know of for the derivative of a function?" Or, in a discrete mathematics and introduction to proofs course, one of my two reading questions might be: "Make up two great examples of your own of multiply quantified statements, in which the meaning changes dramatically when the order of the quantifiers is changed as in Examples 2.2.1 and 2.2.2. Explain why this is the case for each."

So will students read and get much out of it? My experience in many courses and levels is an absolute yes, provided I plan it well. Students commit seriously to reading and writing when they both experience the benefits and know it is highly valued, i.e., in class and in their grade. I mark each reading/writing assignment very quickly, holistically, with a single +,✓,- grade, only for seriousness of effort. I make as many or as few comments on their papers as I wish or have time for, requiring only about five to fifteen seconds per paper, since I am never reading detailed mathematics. My greatest intent is to make sure that each student sees that I have read and thought about what they wrote. Students become very faithful to this reading and writing, and although I expect less than half a page of response, some students become so emphatic about its benefits that they insist on writing more, whether I want it or not! Some even explicitly credit their success in the course to this activity.

My goal for student reading and writing is always two-fold. First, students should engage in meaningful first contact with new material on their own, and second, I need their answers to my reading questions, and their own written questions about the reading. If I receive their responses up to one class period beforehand, I can read them and make some notes enabling me to know how my students are reacting to the new material, and best preparing me to guide class without even any nagging impulse to lecture. I spend no time preparing a lecture, rather I prepare notes on their writing so that I can best guide their learning in the classroom.

Does this require reading material different from a textbook? Not necessarily, provided the reading is genuinely accessible, interesting, stimulates provocative thinking and questions on first contact with new ideas, and provides good grist for class discussion. So I choose reading materials carefully for these goals, possibly utilizing multiple materials with different points of view to compare. This does not mean that I choose material that promises to make the subject "easy" or a "straight path", since such features may mean that the challenges, questions, interest, and depth are missing, which does not serve students in the long-term.

In-class discussion of reading/writing

Class can now begin with a discussion closely guided by me, based on the few notes I made while reading students' responses. It can be truly brief, since it is always focused just on their writing, instead of a shoot-in-the-dark first-contact lecture trying to address everything without knowing what students are struggling to understand.[7] I may choose particular reading responses to have individual students read out loud as part of guiding discussion. The new paradigm has extended to *You-We-...* . Since students have thoughtfully engaged the reading, this second contact in-class discussion never needs to be lengthy, usually 5-15% of class time, and the vast majority of time is available for something else. What should it be?

Warm-up problems beforehand

Auspiciously, student written response to first-contact reading has prepared them for productive initial mathematical problem work. So why not assign easy-to-medium-difficulty warm-up problems also to be prepared in advance and brought to class? In class these problems can be compared, discussed, presented, completed, and generally beaten to death, using the vast majority of classroom time, so that by the end of class the level of student mathematical accomplishment, and their confidence, is high.

Will students actually prepare problems on new material before class? My experience there, too, is an unequivocal yes, provided I design well. It is still crucial that students experience the benefits, know it is highly valued, and be fully expected to contribute in class. This has been so successful that I have never had a single student express reluctance about doing this homework as preparation before, rather than after, class. The learning benefits quickly become obvious to students, since by the end of class time, they are confident they have solved the easy-to-medium warm-up homework, and feel ready to tackle some harder problems at home. The paradigm has further enlarged to *You-You-We-...* .

In-class work on warm-up problems

Primary in-class activity utilizing the prepared warm-up problems takes continually varying form. When I arrive at the appointed hour, I usually find most students already comparing their prepared work in small groups, and often much class time is spent this way, as I continually circulate to discuss with students, my aim being to interact personally with every student or group multiple times. My own classes have ranged from 10 to 50 students. Even in a class of 50, I am usually able to interact personally and substantively with every student at least once during a 75 minute class period. I keep on the move, staying with each student or group just long enough to provide encouragement, a little advice, and to learn what they are struggling with. I spontaneously initiate either whole-class discussions on particular problems, or individual student board presentations and discussion. Often I will ask several students to write solutions to various problems on the board simultaneously,

[7] Lecture has a shoot-in-the-dark character because it is a one-way communication; its origin is 'to read a discourse aloud'. It is interesting that the 'co' in communication (etymology 'sharing', Latin *communis*, 'common') seems absent in lecturing, so perhaps lecture doesn't deserve to be called communication.

and then we discuss them all at once as a class. Sometimes a writer is asked to verbally explain what has been written, sometimes not. Not every problem necessarily gets presented or discussed.

What are the forces ensuring that students really prepare problems before class? Partly it is group peer pressure, and subtle pressure by me circulating to observe student prepared work, and also the certain knowledge that I may ask any student to present a prepared problem on the board at any time. I make my presentation choices of problems, and by which students, on the fly, but very carefully and consciously, including to apply pressure for better advance preparation by individuals if necessary. But students' main motivation to prepare is their experience that it creates a very effective learning environment, one in which they will end class well equipped for the final, harder, after class homework.

I collect the warm-up problems at the end of class, and mark them holistically +,✓,-, again strictly for seriousness of effort at advance preparation, and they are important in the grade too. I could alternatively have a copy due at the beginning or before class, if necessary. Since the warm-up problems have been dissected in class, I never read these problems individually. I am interested solely in whether the student prepared them in good faith before class. This literally takes only five seconds per paper. Even though I am collecting them only at the end of class, it is not hard to train myself to instantly assess preparation in advance. This is particularly easy if, as often happens, there is a warm-up problem that we didn't get to in class discussion; then I can easily see on each student's paper whether they prepared the problem beforehand or not.

You-You-We-You

With the advance preparation of reading/writing, and of warm-up problems, in support of in-class discussion, group work, and presentation, it remains only to assign a very few (two or three) harder homework problems for students to complete after class. These are like harder traditional homework, but build on what has been achieved before and during class, and the full paradigm matures to *You-You-We-You*. These few "final problems" are the only papers needing detailed marking, representing each student's highest level of achievement on the material. They receive careful feedback, a single holistic letter grade for each paper[8], and possible redoing of individual problems at my initiative to bring to perfection. They are normally never discussed in class. These higher level problems are at the core of a student's course grade; I consider them the best measure of what each student has learned and accomplished. The message to students is that their daily *You-You-You* written components are the fundament of both learning and evaluation, so I find it critical that they form the vast majority of the course grade, with exams much reduced. I have always made the three components together of student daily work at least 60% of the holistically assigned course grade, with the final problems dominant in my mind. Since almost all of the reading/writing and problem preparation assignments earn a +, the harder, after class, final problems become paramount.

[8]I have long found using points in marking to be a time-wasting, exhausting, distracting, and deceptive morass that sends the wrong message to students and invites trouble. While I may write a lot on student papers, I always assign only a single holistic qualitative evaluation to a paper, be it homework or an exam.

A word of warning based on experience: Once upon a time I didn't clearly separate the warm-up from the final problems, but this led to lots of complications, including lesser student effort on the warm-up problems before class; I find it works far better with the two sets well separated.

How can one be sure that these final problems completed outside class represent the work of each individual, especially since I encourage students to work together in class so much? Truth be told, even on the final problems I make it clear that students may discuss them together. My ironclad rule, though, is that when they go to write them up, this must be done alone, so that no two papers should look alike. Since these harder problems are never just a calculation or something formulaic, but always involve explanation of ideas, I can always detect not only the level of understanding, but also easily observe if two papers are too similar. If so, which occasionally happens at the beginning of the course, I speak with the students involved to reiterate my expectations as emphatically as necessary.

Précis

To sum up the evolved paradigm *You-You-We-You*, students write three homework papers for each class day, which I call parts A,B,C:

- *You*, Part A: Read/write, received early for me to prepare for leading class discussion. Marked quickly +,✓,- for effort only.
- *You*, Part B: Warm-up problems, prepare for class. Received at end of class, marked quickly holistically +,✓,- for preparation effort only.
- *We*: In-class discussion, group work, presentations, all based on Parts A,B.
- *You*, Part C: A very few harder problems, completed after class and written up alone. Marked carefully with feedback and holistic letter grade, sometimes specific problems redone at my request.

Together Parts A,B,C constitute the majority of the course grade, reducing exams.

Note that for a given class day, the Parts A,B,C due are for three different units of material, so there is a rolling nature to the coverage of three units. Students easily adapt to this, and it has an integrative benefit.

Inertia

There are many forces locking an instructor into the *I-You* paradigm, even if open to change. First, we tend to teach as we were taught. Second, a lecture "covering material" provides a seductive feeling of fulfillment of professional duty, and feels like an insurance policy against any responsibility for student failure to learn. Moreover, a "professing" of one's authoritative knowledge of the subject is often enjoyable. It is hard to let all these go, and to realize that instructor coverage does not necessarily help students learn. Students, too, are mostly happily complicit, generally very comfortable with passive receipt of a lecture. Certainly it is much easier than having to do any actual work in the classroom, and they can believe

they must have learned something from lecture, even if there is no evidence for it. Third, it takes real effort to change pedagogy, probably requires more time at the first pass, and will likely catalyze a process of further evolution, so is a major commitment. Fourth, there is an element of uncertainty, worry, and perhaps fear of classroom disaster. Lecture is a well-known, often easy, predictable certainty, containing essentially no element of risk, since it is totally controlled by the instructor based on preparation in advance, with little chance of anything, let alone something unexpected or surprising, from students. On the other hand, a classroom of constant interaction with students, and continual reacting to, adjusting, and guiding of what students initiate may seem scary. Moving away from lecturing amounts to relinquishing total control, but hopefully without totally losing control, since one still has overall guiding responsibility. Creating the right balance is a challenge.

Therefore, since shifting from *I-You* requires overcoming much inertia, it will most likely occur only if one sees huge benefits and rewards, and not too many scary challenges, not only for students and their learning, but also for instructors themselves, and here I would like to offer much encouragement from experience. Let's begin with the students.

Benefits and challenges for students and learning

My experience is that students buy strongly into preparatory work provided it is designed so that they quickly and consistently experience the advantages, and that they will find their in-class work time valuable, engaging, rewarding, often exciting, and confidence-building. Completing the warm-up problems with feedback from me and fellow students in class prepares students well for success with the few final, harder problems to be completed after class, and they know and greatly value that. I have many times had the experience that my students are so absorbed in group work completing warm-up problems that they don't realize when class has ended. I apologetically interrupt the whole class to tell them that class ended five minutes ago! When does that ever happen with a lecture?

I discovered painfully once, in a beginning calculus course, what happens if I steal their in-class work time by lapsing into lecture. I thought that the material for the day was particularly tough, and that if I began with a bit of lecture, it would help. After a while I saw frustration, even anger, on my students' faces, and I realized my mistake: They wanted to get to work on what they had prepared for their valuable in-class time together, not listen to me steal their work time. I now realize I should be very happy about that; they are in charge of their learning, and they know it and own it.

The reduction of exams along with the predominant emphasis on daily work for both learning and course grade creates a much steadier workload for students, yielding the cognitive advantages of spaced learning, and relief from the typical exam-cram-forget phenomenon that doesn't foster long-term learning. This also places learning and evaluation in harmony, reducing stress and producing more consistent quality of work. My impression from a 20+ year evolution is that with these approaches my students work more, and more effectively, and that they more successfully learn course material.

Student course evaluation comments are almost uniformly positive about the pedagogy. Their comments typically credit advance preparation for in-class collaborative work as extremely effective for their learning, and for keeping them on top of the course with less stress. Students also comment on how the emphasis on student participation makes the

subject come alive. And quite frequently they ask why other mathematics courses are not taught this way.

I mention here one anecdote that still astonishes me, from an abstract algebra course intended both for mathematics majors and future secondary mathematics teachers. Although the entire course was focused on mathematics, at the end of the semester one student came to my office to tell me that for her, more important than the mathematics had been the teaching style, and that she had consciously spent the entire term studying the pedagogy, with the aim of adapting it in her own teaching. Never had I dreamt that while thinking I was teaching abstract algebra I was actually also teaching pedagogy.

Challenges for instructors

Shifting from *I-You* to something like *You-You-We-You* has initial challenges for an instructor. As with anything new, more effort will be needed the first time through. Experience pays off handsomely, though, and after once or twice through, the overall workload should be no greater than for *I-You*, and the rewards are substantial.

It is critical that students buy in from the start. I tell students that it will enable them to be successful in the course, that class time will be interesting, productive, and satisfying, that it will prepare them well for the harder homework, and that this daily work is the great majority of their course grade. I aim to build confidence in students from the very start, and tell them I will be there to give personal help in class every day. Then I watch and listen to how things are going, especially in the first weeks, and take steps to resolve confusion and alleviate discomfort.

One must learn how to challenge and help students in the right way in the classroom, and to constantly manage and make decisions in an unpredictable classroom environment where control and responsibility is being loosened and partially handed to students. At every moment I am thinking not only about the student I am working with on mathematics, but also about what I should do next. Should I ask a particular student to put a certain problem on the blackboard, or initiate a whole-class discussion on a particular problem, or go on to another student or group? Compared to this, lecturing is easy.

I need to keep reminding myself that in a nonlecture classroom, it is students who should be doing the mathematics, not the instructor, since I already know the mathematics, and they are the workers and learners. My job really should be that of guide and manager.[9] Neither should anyone expect their learning to be easy: I can be helpful in many ways, but the learning is their work, just as when Euclid is said to have replied to King Ptolemy's request for an easier way of learning mathematics that "there is no Royal Road to geometry". Without meaning to sound flippant, my job should be to guide students to do as much useful work as necessary to learn, while doing the least of it myself.

Perhaps the greatest danger for an instructor is that with students handing in homework Parts A,B,C for each class day, it would be all too easy for me to do way more homework marking than I should, and therefore spend more time teaching this way; I have witnessed colleagues insistently fall into such a hole when trying this approach. Each of the three parts

[9] Frank Williams was a seminal influence for me, for which I am forever grateful. It was he who helped sink home in me the proper role for the instructor to complement the student's role of worker and learner [26].

is crucial for students to be doing, but Parts A,B do not need grading or instructor feedback on the mathematics, since this all happens in class. My grading of Parts A,B is extremely brief, only a few seconds per paper. While Part C is carefully marked, and perhaps parts redone, the key is that it should only consist of two or three of the harder problems, never more.

Is this a one-size-fits-all approach? While I have found the basic components to be universally successful, the details may best differ between courses at different levels. For instance, in a mathematics appreciation general education course at the lowest college level, I emphasize hands-on activity more than reading, for both work at home and in class; and after class work often involves just writing up what they discovered in class. At the other extreme, in a Ph.D. level graduate course, I often ask students to contrast multiple different written approaches to the mathematics, and in class I will ask students to present proofs at the board and lead discussion thereof.

Benefits and rewards for instructors

Perhaps for many instructors, at the end of the day it will be the personal rewards, not just those for students, that will seem attractive about alternatives to *I-You*. I admit I have reaped tremendous personal rewards.

Class time has higher quality interactions and is more exciting when one is constantly discussing interesting mathematical ideas with individuals and groups, and they are coming up with questions and ideas and points of view that one hadn't anticipated. The enthusiastic response from students is extremely gratifying, as is the learning success one sees. In short, I enjoy interacting with my students much more, a huge benefit!

Marking student work is more rewarding in two senses. First, exams are fewer. Second, marking time is spent primarily on the few harder homework problems, which are more interesting to mark, not on the easier material that has been dealt with in class. And the remaining time is spent mostly reading student responses to their first-contact reading, which is interesting and prepares one with confidence to lead a class discussion most useful for student learning.

Time, ah time: My experience over many courses is that an alternative to *I-You* need not take more instructor time overall, provided one does not fall into the trap of unnecessary over-marking of student papers discussed earlier. I do not spend more time on teaching than I did before. A perhaps surprising timesaver is that students often need less of my time in office hours. By replacing lecture with student interaction with each other and with me on active work in the classroom, students get most of the help they need, and their questions answered, in class. Moreover, the steadier workload mentioned above applies to instructors as well, so there is very little end-of-term stress, and in the long-term less burnout.

With rewards as strong as these, I could never return to *I-You*.

Is there really an elephant in the classroom?

Finally, consider the question of coverage, an intimidating and much-feared elephant. When I talk with *I-You* instructors about replacing lecture with student work in class, they almost invariably reply "But then I couldn't cover all the material in the syllabus". My primary

answer, of course, is that it is not the instructor who needs to cover the material, but rather the student.

This leads to multiple possible views on the purported elephant in a student-centered classroom. One is that even though startup is slower, more is covered in the end, because coverage becomes more efficient. Another view is that even if less material is covered, this is compensated for by students' developing ability to think mathematically and be self-directed learners. A third is that even if less material is covered, students retain much more with active classroom spaced learning than with the traditional lecture/notes/examcram format.

My personal experience has been different from all these conceivable elephants. In teaching many types and levels of courses I have found that if high quality first contact and initial mathematical work happens before class, making lecture irrelevant and redundant, and if class time is instead used for student work with others and with the instructor to build on the work prepared in advance, then coverage is always more efficient, not less so. To me this simply makes logical sense: If lecture is largely an inefficient use of precious classroom time for student learning, then offering students a guided active learning classroom environment with each other and with me seems likely to proceed more efficiently, especially when first-contact reading and preparatory work happens before class. Specifically, I have taught this way in first year calculus courses with multiple sections all following the same lockstep routine with common exams, where students in my section had to progress in class at the same rate that other instructors were lecturing, and this was no problem at all. In fact it was in exactly that setting, with a class of 45 students and no grader or teaching assistant, where I first developed and refined the approach described here.

My consistent experiences after transforming *I-You* into *You-You-We-You*, in many courses at all levels and for all college audiences, is that the content is less rushed. I found no fearsome coverage elephant in the classroom as I redesigned it, even with the same syllabus as other instructors.

The takeaway

I evolved away from lecture by integrating the ingredients of student advance reading/writing and problem preparation with the active learning prescription of student collaborative classroom work and instructor feedback, all followed by higher level homework after class. These daily activities form the core of my students' work and evaluation, and I have found over many courses that students can thereby have a rich and successful learning experience in which they are confidently in charge of their own learning. Moreover, since class time is spent with students working, rather than me lecturing, their coverage of a syllabus is better.

What must I at minimum have within my control in order to teach this way? I need my students to have access to good reading and problem material that I can assign as needed, including reading/writing and problem preparation in advance of class. I need the daily pre- and post-class assignments to be the core of students' work and grade. And I need to be able to mold the classroom environment into an active one and gain the confidence of my students. All else is flexible.

As instructor, there are many personal benefits, perhaps the happiest being much more rewarding, interesting interactions with students, and witnessing their successes, confidence, and satisfaction. Teaching this way does not take more time, provided I do not spend more

time marking papers than my students need as feedback on out-of-class work, and that I always remember that they are the workers whose job is learning, with me merely an effective, efficient and encouraging guide.

So I see rewards for all in the shift from lecture to active learning. Carpe diem.

Acknowledgements: I am indebted to Virginia Warfield, Barbara Walvoord, and Frank Williams for critical insights along my journey, and to Pat Penfield and many others for encouragement and constructive criticism.

References

[1] C. Brame, *Flipping the classroom*, Vanderbilt University Center for Teaching, 2013. Retrieved Sunday, March 1, 2015 from `http://cft.vanderbilt.edu/guides-sub-pages/flipping-the-classroom/`.

[2] B. Braun, P. Bremser, A. Duval, E. Lockwood, D. White, What does active learning mean for mathematicians?, *Notices of the American Mathematical Society* **64** (2017), 124–129, `http://www.ams.org/publications/journals/notices/201702/rnoti-p124.pdf`.

[3] D. Bressoud, The Worst Way to Teach, *Launchings*, July 2011, Mathematical Association of America, `http://launchings.blogspot.com/2011/07/the-worst-way-to-teach.html`.

[4] D. Bressoud, Evidence of Improved Teaching, *Launchings*, October 2013, Mathematical Association of America, `http://launchings.blogspot.com/2013/10/evidence-of-improved-teaching.html`.

[5] Conference Board of the Mathematical Sciences, *Active Learning in Post-Secondary Mathematics Education*, July 15, 2016, `http://www.cbmsweb.org/Statements/Active_Learning_Statement.pdf`.

[6] J. Epstein, The Calculus Concept Inventory—Measurement of the Effect of Teaching Methodology in Mathematics, *Notices of the American Mathematical Society* **60** (2013), 1018–1026, `http://www.ams.org/notices/201308/rnoti-p1018.pdf`.

[7] S. Freeman et al, Active learning increases student performance in science, engineering, and mathematics, *Proceedings of the National Academy of Sciences* **111** (2014), 8410–8415, `http://www.pnas.org/content/111/23/8410.abstract`.

[8] E. Green, Why Do Americans Stink at Math?, *New York Times*, July 23, 2014, `http://www.nytimes.com/2014/07/27/magazine/why-do-americans-stink-at-math.html`.

[9] R. Hake, *Can the Cognitive Impact of Calculus Courses be Enhanced?*, 2014, `http://www.physics.indiana.edu/~hake/ImpactConceptInventoriesB.pdf`.

[10] *Just in Time Teaching*, 2006, `http://jittdl.physics.iupui.edu/jitt/`.

[11] M. Kogan, S. Laursen, Assessing Long-Term Effects of Inquiry-Based Learning: A Case Study from College Mathematics, *Innov. High. Educ.* **39** (2014), 183–199, http://link.springer.com/article/10.1007/s10755-013-9269-9.

[12] Y. Lai, *Teaching Undergraduates Mathematics – MSRI Workshop Report*, Critical Issues in Mathematics Education Series, Volume 5, May 2009, Mathematical Sciences Research Institute, http://www.msri.org/attachments/TUM-manuscript-030612.pdf.

[13] C. Lambert, Twilight of the Lecture, *Harvard Magazine*, March-April 2012, http://harvardmagazine.com/2012/03/twilight-of-the-lecture.

[14] S. Laursen, M. Hassi, M. Kogan, A. Hunter, *Evaluation of the IBL Mathematics Project: Student and Instructor Outcomes of Inquiry-Based Learning in College Mathematics*, 2011, 'at http://www.colorado.edu/eer/research/steminquiry.html.

[15] S. L. Laursen, From innovation to implementation: Multi-institution pedagogical reform in undergraduate mathematics. In D. King, B. Loch, L. Rylands (Eds.), *Proceedings of the 9th DELTA conference on the teaching and learning of undergraduate mathematics and statistics*, Kiama, New South Wales, Australia, 24-29 November 2013. Sydney: University of Western Sydney, School of Computing, Engineering and Mathematics, on behalf of the International Delta Steering Committee, 2013, at http://www.colorado.edu/eer/research/steminquiry.html.

[16] E. Mazur, *Peer Instruction*, 2015, http://mazur.harvard.edu/research/detailspage.php?rowid=8.

[17] National Science Foundation, *Enough With the Lecturing: Active Learning Improves Grades, Reduces Failure Among Undergrads in STEM*, Press Release 14-064, May 12, 2014, http://www.nsf.gov/news/news_summ.jsp?cntn_id=131403.

[18] G. Novak, E. Patterson, A. Gavrin, A, W. Christian, *Just-in-Time Teaching: Blending active Learning and Web Technology*, Saddle River, NJ: Prentice Hall, 1999.

[19] D. Pengelley, Beating the lecture-textbook trap with active learning and rewards for all, *Notices of the American Mathematical Society* **64** (2017), 903–905, http://www.ams.org/publications/journals/notices/201708/rnoti-p903.pdf.

[20] SCALE-UP: Student-centered active learning environment with upside-down pedagogies, http://scaleup.ncsu.edu/

[21] B. Walvoord, V. Anderson, *Effective Grading: A Tool for Learning Assessment*, Jossey-Bass Publisher, San Francisco, 1998.

[22] V. Warfield, Email Newsletter #40, February 27, 1998.

[23] K. Weber, Mathematics Professors and Mathematics Majors' Expectations of Lectures in Advanced Mathematics, *On Teaching and Learning Mathematics: AMS Blog*, American Mathematical Society, 2015, http://blogs.ams.org/matheducation/2015/02/10/mathematics-professors-and-mathematics-majors-expectations-of-lectures-in-advanced-m

[24] C. Wieman, Large-scale comparison of science teaching methods sends clear message, *Proceedings of the National Academy of Sciences* **111** (2014), 8319–8320, `http://www.pnas.org/content/111/23/8319.extract`.

[25] C. Wieman, A Better Way to Evaluate Undergraduate Teaching, *Change*, January/February 2015, 6–15, `http://www.changemag.org`, or `http://www.cwsei.ubc.ca/SEI_research/index.html`.

[26] F. Williams, Total Quality Management in the Classroom: Applications to University-level Mathematics, *Intl. J. of Mathematical Education in Science and Technology* **26** (1995), 743–747.

David Pengelley is professor emeritus at New Mexico State University, and courtesy professor at Oregon State University. His research is in algebraic topology and history of mathematics. He develops the pedagogies of teaching with student projects and with primary historical sources, and created a graduate course on the role of history in teaching mathematics. He relies on student reading, writing, and mathematical preparation before class to enable active student work to replace lecture. He has received the MAA's Deborah and Franklin Tepper Haimo teaching award, loves backpacking and wilderness, is active on environmental issues, and has become a fanatical badminton player.

Department of Mathematical Sciences, New Mexico State University, Las Cruces, NM 88003
Department of Mathematics, Oregon State University, Corvallis, OR 97331
davidp@nmsu.edu