Final Report & Reflection

of

Picacho Middle School’s
Spring 2010
Math Lesson Study Team
# Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Topic</th>
<th>Page Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Overview</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Team Reflections</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>Individual Reflections</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Rey’s Reflection</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Tom’s Reflection</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Doug’s Reflection</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Abram’s Reflection</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>Lesson Plan</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>Record of Team Member 2010 Accomplishments</td>
<td>21</td>
</tr>
<tr>
<td>6</td>
<td>Record of Meetings, Topics, and Progress</td>
<td>21</td>
</tr>
</tbody>
</table>
Section 1:
Overview

Picacho Middle School Lesson Study Team Members:
   Doug Lutz
   Rey Munoz
   Linda Prendez
   Abram Sanchez
   Tom Taney

Rey, Abram, and Tom are math teachers at Picacho Middle School in Las Cruces, New Mexico. Rey teaches 7th grade math and is also the math department chairman. Abram teaches 6th grade math. Tom teaches 8th grade math and algebra I. Linda is the Professional Development Teacher at Picacho Middle School. She is a full-time professional developer and no longer teaches students. Doug teaches math and engineering at Las Cruces High School in Las Cruces, New Mexico. He is also the math department chairman at LCHS.

Our team formed in December of 2009 and had our initial meetings in January 2010. We met weekly after school on Mondays with a few exceptions. Doug had participated in two lesson study groups at Las Cruces High School in the past and had participated in the lesson study trainings associated with those lesson study groups. The trainings were led by Aki Takahashi, a professor at DePaul University. Rey and Abram had participated in one lesson study in the past. That lesson study was organized by Ted Stanford, a math professor at New Mexico State University. Tom had attended training led by Aki Takahashi about lesson study, and had viewed 2 public lessons and de-briefs in the past, but had never participated in a lesson study team himself. Linda had never participated in a lesson study, nor had she attended lesson study training before joining this lesson study team.

This lesson study was conducted in association with the Building Lead Teachers Grant from New Mexico State University (NMSU). Both Doug and Tom are teachers in the grant and were co-leaders of the lesson study team. The grant chair is Susana Salamanca, a math professor at NMSU. Susana attended several team meetings and practice lessons. She is mentioned in this report.

There were three other lesson study teams associated with the Building Lead Teachers Grant. Those teams were at Santa Teresa Middle School, Chaparral Middle School, and at Desert Trail Elementary School of the Gadsden School District in southern New Mexico. The members of this lesson study team attended training with the members of the other teams and viewed their public lessons. The training for this grant was conducted by Blake Peterson, a professor at Brigham Young University. Blake and some members of the other lesson study teams are mentioned in this report.
Section 2:
Team Reflections of the Picacho Middle School
Spring 2010 Math Lesson Study Team

Team Reflection Topic # 1:
Did it make a difference that our public lesson was taught “out of the blue” instead of being taught as part of a probability unit?

Our Lesson was originally designed to be the third lesson taught in the first investigation of a 7th grade probability unit. We changed the original lesson for two reasons. First, we did not feel we had enough time to have the students roll dice to conduct the experiment in the same class period that they explored the question of the theoretical probability associated with this lesson. Picacho Middle School operates on a 7 period day with 52 minute class periods. Secondly, the lesson as written did not address the law of large numbers, and a student understanding of the law of large numbers was one of our lesson goals. Rey Munoz taught the first round of practice lessons in his 7th grade classes. Rey taught the lesson as we planned it, but he taught it in place of the original lesson. Rey’s students were exposed to this lesson as part of a probability unit. They had done a lesson dealing with tree diagrams the day prior to this lesson. They studied area models of probability the day after this lesson. Our second practice lesson and our public lesson were taught in two of Tom Taney’s 8th grade classes. In both classes, the lesson was taught “out of the blue”. The students were not studying probability when they were exposed to this lesson.

Students in Rey Munoz’ classes tended to get started on the exploration quicker than students in Tom Taney’s classes. Also, students in Rey’s classes tended to come up with a larger variety of solution methods than did students in Tom’s classes. In the second practice lesson, the students in Tom’s 3rd period class all used the same solution strategy, an organized list. No students tried area models or tree diagrams. Additionally, no students attempted an organized list in which they listed products first. Every group used organized lists with factors listed first. In the public lesson, most groups attempted organized lists with factors listed first, but a few groups did attempt organized lists with products listed first, and a few attempted tree diagrams. We are curious if these differences between Rey’s and Tom’s classes were caused by the fact that Rey’s students were exposed to this lesson during a probability unit while Tom’s students were not.

Although the differences between Rey’s and Tom’s classes were interesting, we really have no way of knowing if the “out of the blue” nature of this lesson in Tom’s classes vs. the “part of a unit” nature of this lesson in Rey’s classes was the cause of the differences. We concluded that “out of the blue” vs. “part of a unit” probably contributed to the differences between the classes, but the differences were probably due more to differences in the students in each class and to differences in Tom’s and Rey’s day to day teaching styles and methods.
Team Reflection Topic #2:
Should we have used 36 roles in the experiment the day prior to the lesson?

In the original CMP lesson, students were asked to play the dice product game by rolling two dice 36 times and recording the results. They used these results to get an experimental probability for the dice product game. There are also 36 possible outcomes when rolling two standard dice, a fact which is very important in determining the theoretical probability of rolling an odd or even product with two dice. This coincidence could lead students to confuse experimental and theoretical probabilities. (In fact, we did not observe any students have this confusion during the practice lessons, but several students had this confusion during the public lesson). We had predicted this possible confusion and we did discuss changing the number of rolls for the experiment; however, we chose to leave the number of rolls at 36 for the experiment. We made our decision for two reasons. First, we wanted to bring out this confusion so we could deal with it if it in fact existed. Secondly, we thought the original authors must have written this “coincidence” into the curriculum purposefully and we assumed they were wiser than us.

In hind sight, we think it was a mistake to have 36 rolls in the experiment. Although one of our goals did deal with using experimental probability to evaluate a hypothesis about theoretical probability, that goal did not include a desire on our part to purposefully confuse the differences between experimental and theoretical probabilities so that we could then clarify the difference. As a result of our discussion after the public lesson, we changed the number of rolls in the lesson plan to 30. We chose a number that was not 36. We further chose a number that is not divisible by 4. In this way, none of the group’s experimental data can exactly match the correct theoretical probability of odd (1/4th) or of even (3/4th). We think an exact match could possibly lead to the same confusion between experimental and theoretical probabilities. We think 30 is a large enough number to allow for a good understanding of the law of large numbers when the class totals all their rolls in the summary. We did discuss having 50 rolls but did not think the extra time needed for the extra 20 rolls would necessarily lead to a better understanding of the law of large numbers.

Team Reflection Topic #3:
Is lesson study about coming up with a good lesson?

Some of the members of our group originally thought that lesson study was about the end product. They thought the goal of lesson study was to come up with a good lesson. None of our group members think that any longer. A good lesson is a byproduct of lesson study, but our group members now agree that the important part of lesson study is what you gain and learn in the process. Although each group member got something different from lesson study, each of us now agrees that what we gained from the process is far more valuable than the lesson we developed. Each of our member’s individual reflections follow in a later part of this report, but the one common theme we all agreed upon as a strength of the lesson study process is “student thinking”. Whether we were anticipating it, planning how to direct it, planning a response to it, or simply watching it; we all agree
that the ability to think about “student thinking” afforded by lesson study has made us better teachers.

Team Reflection Topic #4:
Should we have used the word “real” in the question “What is the real probability of rolling an odd or even product on each roll of the dice?”
(The Theoretical Probability)

We thought of the “real probability” as theoretical probability, whereas some of our students may think of “real probability” as experimental probability. We did not even think of this until the de-brief of our public lesson. One observer asked why we had used the word “real” in the question. This question took us all by surprise because we had not even considered the question before. We now agree it was probably a poor choice of words. We have changed the question to read “What is the theoretical probability of rolling an odd or an even product on each roll of the dice?”

Team Reflection Topic #5:
Should we have included a goal about students seeing and understanding various representations of the theoretical probability in this lesson? Should that goal have caused us to introduce representations which students did not come up with in the summary of the lesson?

Susana Salamanca attended our first meeting after our public lesson. She questioned why we ordered our share out methods in the manner we did. This led to a discussion about the questions above. We discussed the importance of sharing various representations in our initial meetings when we were deciding on a goal for our lesson and choosing a lesson to accomplish that goal. We decided that we wanted to try to get to an understanding of the law of large numbers. We purposefully did not include a goal of students seeing and understanding various representations of the theoretical probability in this problem.

Tom tried to introduce a representation which the students had not generated in the summary of the practice lesson in which all his students used the same representation. In the process he lost the time necessary to lead the discussion to the law of large numbers. This event reinforced with the team that our goal was to help students understand the law of large numbers. Our goal was not to have students see and understand various representations of the theoretical probability involved in this problem. We have not changed our minds on this topic. We do not argue the value of understanding various representations; however, we do not believe it must be an underlying goal of every lesson. In particular, it was very purposefully not a goal of this lesson.

We do not picture this lesson as a standalone lesson. We picture this lesson as part of a probability unit. We do intend for all students to see all the representations in this lesson as part of the unit.
Team Reflection Topic #6:
Surprises

As we reflected on the lesson study process we had just went through, several of our thoughts centered on ways in which students and classes surprised us. I listed a few of those surprises below.

In our initial meetings, we all agreed that our students would not come up with the representation of an area model with odd and even listed on each side. Two groups in Rey’s classes came up with that representation of the theoretical probability (one correctly, and one incorrectly).

We also did not think our students would come up with the area model representation with 1 through 6 on each side; however, several groups came up with that representation.

Students did not make several of the mistakes we anticipated during the practice lessons.

Students made several mistakes we had not anticipated like making an organized list but organizing it around the products instead of the factors.

Some topics that were very difficult for students during the practice lessons did not present obstacles to students in the public lesson. The largest of these was the difference between 2*1 and 1*2 and weather these are two different outcomes.

Some lessons learned from our “surprises” are:

- Don’t underestimate your students. They will probably surprise you.
- We are not as good at anticipating our student’s thinking as we probably should be.
- Don’t drop an anticipated student error or method from your plan because it has not occurred in several practice lessons. It may just happen in the public lesson.

Team Reflection Topic #7:
What should we do with students who finish early?

In our original lesson plan, we planned on asking students who finished early to look at their experiment sheet from the day prior and to figure out their experimental probability. We than asked them to start thinking about their experimental and theoretical probabilities and try to explain why they were the same or different. In the public lesson de-brief, Blake Peterson asked if there was possibly a way to push students who finish early to come up with the odd-even explanation of theoretical probability by telling them that there was a way to come up with the theoretical probabilities without using 36 and then challenging them to see if they could figure it out.
Our original plan simply tried to take students who finished early and start them thinking about the main idea of the summary a little before the other students. Blake’s plan for students who finished early pushed the students into thinking about this problem in a more general and mathematically elegant manner. After discussing these two options of how to deal with students who finished early, we think that attempting to push bright students to a more general and elegant manner of thinking about mathematics is preferable to simply pushing them ahead of the other students in the class. As a result, we changed our strategy for dealing with students who finish early. We went with Blake’s strategy and question.

Section 3:
Individual Reflections of the Picacho Middle School Spring 2010 Math Lesson Study Team

Individual Reflections of Rey Munoz

Rey’s Reflection Topic #1: The Importance of thinking deeply about “Student Thinking”

One of the things I learned from this lesson study group this semester was how important it was to take the time and think about "student thinking." And not to stop thinking about it during responses at the beginning of the lessons, but to also think about it during the lessons and to continue to think about it after the lesson is taught. Another important aspect of doing this is also the importance of doing it together as a team.

Many rich discussions and ideas came out when, as a group, we talked about the different strategies, questions, or problems that the students might have. For example, in our initial planning, there was talked about how the students might come up with the theoretical probability of the game by just using number theory. This whole idea of them being able to find the theoretical probability this way, first of all, did not cross my mind, much less my students being advanced enough to be able to use this strategy. When we did the practice lessons in my class, sure enough, some of the students started to think like this and because we had talked and planned about this before hand, I felt I was that much more prepared to answer any questions or understand what the students were trying to do in answering the question this way. It also lead me to understand one students misconception about (2,1) and (1,2) which something we talked quite a bit about in the planning. But without understanding this strategy, I probably would have been lost in trying to understand why the student was getting 1/3 instead of 1/4 for the theoretical probability of getting an odd number. So not only did I find it was important to try to
understand what students might be thinking as we planned the lesson, but to also give my students to benefit of the doubt in that they can and will think higher then you sometimes think they can.

Rey’s Reflection Topic #2:
The Importance of purposefully choosing the order of student “share outs”

Another important thing I have learned during this process that I can apply to my daily instruction is being purposefully on the order of sharing out. Personally, I understand that an area I need to get better at in my own practice is the summary component of my class. I have spent so much time trying to get better at the launch and explore part of our program that I have spent little effort in perfecting the summary aspect. One of the ideas that came to me during our process is this idea of purposefully picking an order to share out. This was done, because we wanted the students to see these strategies as increasingly advanced thinking. In the past, my strategy for share out was either just done randomly or by someone who wanted to share what they had. I learned that having a purposefully share out where ideas or concepts could be built upon each other seemed to be key in helping students with most of the goals of the lesson. I have already begun to think about this in other lessons, the order in which I wanted students to present what they had. This is just one more tool that I have learned that can, and should, make the summary portion of my class that much stronger.

Individual Reflections of Tom Taney

Tom’s Reflection Topic #1:
Lesson Study’s Ability to Provide Different Things to Each Participant

I believe the greatest strength of lesson study is its ability to help each participant grow in an area where they are ready to grow. Each participant in a lesson study gets different things from it. It is not a form of professional development, like many others, in which you learn about a given topic and once you know it, there is nothing else to learn. One teacher may learn more about the misconceptions his or her students bring to a given topic. Another teacher may learn something new about the math they are teaching. Yet another teacher may be exposed to an entirely new or different method of teaching (for example, a traditional teacher may be exposed to a constructivist teaching method for the first time). Another teacher may pick up very specific things like a new method of organizing student notebooks. Kim, the leader the Santa Teresa lesson study team, has participated in four or five lesson studies. She says she learns something new, something different, each time she participates in a lesson study. Very few forms of professional development have the ability to provide something new and different to experienced teachers each time they participate in it. Lesson study, as a form of teacher professional development, has the distinct ability to provide something useful to each of its
participants, even if they are a very experienced teacher who has participated in many lesson studies before. As you read through the individual reflections of the members of this team, you will get a sense of the different things each member took from the process. Because of this knowledge, I am actively trying to promote lesson study as a form of professional development in my school’s math department. We (Picacho’s math teachers) are going to have a lesson study team meeting each semester next year. We have written lesson study into our school’s improvement plan for next year. I plan on participating in both lesson study cycles next year and leading one of them. Rey has volunteered to lead the other cycle.

**Tom’s Reflection Topic #2:**
**Pre-planning teacher reactions to student actions improved my teaching**

I had been out of the classroom for three years prior to this school year. I was employed by a local university and was involved in teacher professional development. During those three years I first learned of constructivist teaching methods. I was immediately drawn to them, but I felt less than authentic when modeling and advocating them. I had never taught in a constructivist manner. I am confident I would have if I had known of it, but I did not. Therefore, I returned to the classroom this year and am struggling with some constructivist methods. One of my greatest faults as a constructivist teacher is that I tend to “tell” too much while helping a group of students. I also tend to stay at any given group for too long while providing that help. I taught one of the practice lessons, and the public lesson for this lesson study. The in-depth planning we did to predict student actions and then to pre-plan teacher reactions made me a far better constructivist teacher. My job was greatly simplified when I approached a group. I was simply trying to see where they were at: down which predicted path were they heading, how far were they, and were they having enough difficulty to cause me to use a pre-planned reaction. I was able to “diagnose” where a group was and to then to take an appropriate action much more quickly. I was able to simply pass by some groups, for they were doing fine. I had a better idea of what all groups were up to and I was far more efficient in monitoring and guiding groups. Students seemed far less frustrated. They were not waiting on me as long as they were accustomed to waiting. I will not be able to put the time into predicting and pre-planning my reactions that the lesson study team did for this lesson, but I can predict some obvious manners in which students will deal with the math of any given lesson and pre-plan my reaction to the pitfalls of that method. I plan on doing exactly that next year. My students and I should both be less frustrated at the end of any given period.

**Tom’s Reflection Topic #3:**
**Lesson study causes teachers to use a reverse planning model**

One of the greatest struggles I encountered as a professional developer was getting teachers to use a reverse planning model like the one advocated by the authors of *Understanding by Design* (UbD). Many teachers blindly follow “the book” or “what they have always done”, and do not think about the big ideas they are teaching in advance. In
the model advocated in UbD, teachers should first decide what students should know and be able to do after the unit. Second, teachers should design the assessment to determine if students know and can do those things. And finally, teachers should plan the unit. Many, if not most, American teachers do the reverse of this process. They pick up a book, be it a traditional or a constructivist book, and “poof wah-lah” their unit is planned. They then grab an assessment designed by the publisher or hastily write their own assessment after they have taught the unit, not before. They may, or may not, ever consider what their students should know and be able to do until they write or give the assessment. They blindly follow “the book” and let its authors do their planning for them.

The lesson study process causes teachers to consider what their students should know and be able to do prior to planning the lesson. It also has teachers consider how the lesson fits into the unit as a whole. The process does not cause teachers to design the assessment prior to teaching the lesson, nor does it cause teachers to reverse plan the entire unit. So, it does not force the entire reverse planning process, but it does cause teachers to decide what students should know and be able do prior to planning a lesson. The lesson study process does a better job of encouraging reverse planning than any other professional development method I have been associated with.

**Individual Reflections of Doug Lutz**

**Doug’s Reflection Topic #1:**
This Lesson was Difficult and Abstract for the Students

My reflections on the lesson and on student learning are that the lesson proved to be difficult and abstract for the students. What I saw in our public lesson was that many students did show all the outcomes for 36 rolls however there was a struggle with understanding that rolling two fair die with the outcome of a 1 and a 2 is not the same as an outcome with a 2 and a 1. Students continually said that they were the same because they have the same product.

Once students successfully determined the outcomes they arranged the products and systematically counted the odds and the evens to show that indeed the “game” was indeed not fair, which contradicted their original hypothesis. I noticed that tree diagrams were not used or any other organizational strategy other than listing the outcomes which for most showed that 27 out of 36 outcomes were odd and 9 out of 36 were even.

**Doug’s Reflection Topic #2:**
Students did improve their ability to use experimental probability data to verify or change their predictions about a theoretical probability; however, they do not gain and understand the theory of large numbers.
Student learning also showed an area of interest to me with very little translation to percentages. Students did not show their outcomes in terms of percentages in that 75% of the outcomes were odd and 25% of the outcomes were even. One student remarked when asked what the percent of the odd outcomes were as “it is hard because it going to be a big number.” It started to become clear that student learning was not making the connect between outcomes and the percentages of odd and even numbers occurring, only listing the outcomes with no connection to our goal was becoming more and more apparent. They did see that their original hypothesis and the use of experimental probability data showed that indeed the “game” was not a fair game, so our second goal, "Students will improve their ability to use experimental probability data to verify or change their predictions about a theoretical probability.” However, our first goal for students to understand the theory of large numbers was not meet due to time, but will make an excellent extension for the revised lesson.

Doug’s Reflection Topic #3: Should we have used 36 rolls in the experiment?

Why 36? Why thirty six was the over arching question during our feedback forum after the public lesson. The process of lesson study involves creating a lesson and generating a goal to develop student understanding. After several practice lessons and a public lesson a round table forum of constructive observation on student understanding takes places to help in the refinement and revision of the lesson. The question why thirty six is about the number of outcomes given two fair dice and was part of the question “is the game fair?” Our rational for using thirty six as opposed to using some other number was first to save time. Our understanding was that in middle school we were greatly limited by a class period of 45 minutes, so in the interest of time we determined the number thirty six was appropriate. In hind-site, as usual, it make perfect sense to present any other number and in fact a different number for each group of students, thus enforcing the outcomes of odd versus even and the law of large numbers.

Individual Reflections of Abram Sanchez

Abram’s Reflection Topic: Strategies I will use in my class because of my lesson study experience

After careful reflection of the Lesson Study I participated in with my colleagues from Picacho Middle School and Las Cruces High School, I have narrowed it down to these strong positive strategies that I will continue to use and implement in my classroom:

- Prior to lesson- Predict student thought processes when solving questions and having detailed questions/thoughts to enhance their problem solving strategies
• Having available enrichment questions to extend student thinking; as well as guiding questions to promote student thinking when having difficulty
• The use of Cornell Notes for student reflection (I saw this while observing the public lesson in Rena’s classroom at Chaparral Middle School)
• “Big Paper” rules and procedures during student exploration (I saw this while observing the public lesson in Kim’s classroom at Santa Teresa Middle School)
• Use of video clips from United Streaming to engage and inform students (I saw this while observing the public lesson in Jaime’s classroom at Desert Trail Elementary School)

I will incorporate these five practices promptly when planning and instructing future lessons. I feel that these approaches will have a positive effect on student education.

However, just the opportunity to participate in professional dialogs about student learning and Mathematical thinking allowed me to grow as a teacher and as a person. By partaking in these rich conversations I became aware little things can make a big difference. Wording, timing, and presentation can make or break a lesson. I have always had an open door policy in my classroom. But inviting colleagues and other professionals in my class to observe and critique student learning is a new notion for me.

Section 4:
The Lesson Plan

Mathematics Lesson Plan for Picacho Middle School
Public Lesson in PMS Room 44 on May 7, 2010

Lesson Planning Team:
Doug Lutz
Rey Munoz
Linda Prendez
Abram Sanchez
Tom Taney

1. Title of the Lesson: Is the Dice Product Game Fair?

2. Goals of the Lesson:
   a. Students will improve their ability to use experimental probability data to verify or change their predictions about a theoretical probability.
b. Students will gain an understanding of the theory of large numbers.

3. Relationship of the Lesson to the Standards (both a 7th and 8th Grade Standard)

   The primary performance objective addressed by this lesson is:

   Strand: Data Analysis and Probability
   Standard: Students will understand how to formulate questions, analyze data, and determine probabilities
   Benchmark: Understand and apply basic concepts of probability
   Performance Objective: Students will use probability to generate arguments, draw conclusions and make decisions

4. Unit Plan

    7th Grade: This is the third lesson of the first investigation in the 7th grade CMP book What Do You Expect: Probability and Expected Value. This investigation is a student’s first experience with the differences between theoretical and experimental probability. This lesson follows lessons that introduce tree diagrams and experimental probability.

    8th Grade: Probability is not normally taught to 8th graders in LCPS; however, most 8th grade teachers do review probability prior to state testing in April. This lesson could be part of that review.

5. Instruction of the Lesson

    6. Rationale for the lesson:

    7. In our initial meeting, we discussed at which math content strands do Picacho Middle School (PMS) students traditionally perform poorly. We looked at NMSBA and MAP scores. PMS students have performed relatively poorly on the “Number and Operation” and “Data Analysis and Probability” strands. We chose this lesson because it involves very important concepts from both strands; the difference between theoretical and experimental probability, the rules of odd and even products, and to a lesser extent, the theory of large numbers.

    The major focus of this lesson:
8. The major focus or theme of this lesson is for students to use their results from a probability game (an experimental probability) to revise or confirm their thoughts on the fairness of the game (the theoretical probability).

9. The above objective will be accomplished by doing the following:
On the day prior to the public lesson, students will record their prediction about the fairness of “The Dice Product Game”, play the game, and record the results of the game. During the launch of the public lesson, students will use the results of their game to revise or confirm their prediction about the fairness of the game. During the explore and share portions of the lesson, students will find the theoretical probability of rolling an odd and an even product in the game. During the summary, students will start looking at the theory of large numbers using their data from the game and their theoretical probability.

The Dice Product Game:
The game is played with two players. Players roll two standard dice. Player A gets a point if the product of the dice is odd, and player B gets a point if the product is even. The players roll the dice 30 times.

10. The Lesson Plan

The lesson plan is on the following 5 pages.

<table>
<thead>
<tr>
<th>Steps, Learning Activities</th>
<th>Teacher Responses to Expected Student Reactions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Day Before:</strong></td>
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<tr>
<td>Students make a prediction about the fairness of the dice product game, play the dice product game 30 times in pairs, and record their results on a data sheet (attached)</td>
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<tr>
<td>• Students turn in one data sheet per pair</td>
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<tr>
<td>• Ensure students are multiplying and not adding as they are playing the game</td>
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<td>• Ensure groups have two different color dice</td>
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<tr>
<td>• Ask the question “what is the probability of rolling a sum of ten on two dice?” (this is the example used on page 5 which we refer students to during the lesson)</td>
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| **Launch:**               |                                                 |
| The teacher rolls the dice once or twice and reviews how the students played the game. |                                                 |
Have students sit with the same partner they played the dice product game with the day before.

Have the Probability Hypothesis Cycle and the Chances of Winning Chart on the board at the beginning of the class. (see the blackboard plan).

As the class starts, the teacher reminds the students of the prediction they made about the fairness of the dice product game the day before. The teacher reminds the students that they predicted a 50-50 chance of either player winning the game. The teacher writes 50% in the Player A column and 50% in the Player B column in the Original Hypothesis line of the Chances of Winning Chart (see the blackboard plan).

The teacher then tells the students that he is going to record all the winners from the game the day before. The teacher then flips through the record sheets from the game that the students turned in the day prior. As the teacher does so, he/she keeps tally of Player A and Player B wins on the Experiment Data line of the Chances of Winning Chart (see the blackboard plan).

Many students will start realizing the game is not fair as the results are tallied and Player B, the player who earned a point for each even product, won all (or most if some teams faked results) of the games.

The teacher then gestures toward the results on the board and asks the question, “Based on the results of our experiment, do you think each player has a 50% chance of winning a point each time the dice are rolled?” The teacher also projects this question onto the board (see the blackboard plan).

The teacher then asks the students to think about the question for one minute and to write down their thoughts. After their minute of private think time, the teacher asks the students to discuss their thoughts with their partner (The teacher allows the discussion to continue for about a minute).

Most students will change their mind about a 50/50 chance of each player winning and realize that Player B must have a greater chance of winning a point on each roll.

Some students may still hold the belief that players A and B each have an equal chance of winning on any given roll, and that the results of the games were simply the result of “luck” or “chance”.

During the discussion, the teacher listens closely for students who believe the results are due to chance.

The teacher then leads a quick (one or two minute) whole-class discussion to bring out the belief that Player B must have a greater chance of winning on each roll based on the experimental data and that the class’s prediction from the prior day must be wrong. (It is ok if the entire class does not believe this at this point; the teacher just wants to get this belief out in the open) The teacher records this opinion in the Revised Hypothesis line of the Chances of Winning Chart (see the blackboard plan).

The teacher then asks, “What is the theoretical probability of Player A and Player B winning a point on each roll of the dice?” The teacher also projects this question on the board (see the blackboard plan).
The teacher reminds the students that they turned in a record of their rolls from when they played the game the prior day. The teacher reminds the students that they can use their experimental results from the day prior or any other method they can think of to help them figure out the chances of each player winning on any given roll of the dice. The teacher has a student hand back the experimental results from the day prior.

**Explore:**
(Give the students 20 minutes and emphasize this)

Students try to figure out the answer to the question, “What are the real chances (the theoretical probability) of Player A and Player B winning a point on each roll of the dice?”

**Some students may not start**

**Some students may list the products first and then list the factors after the products**

- Some students may get confused with 1x2 vs. 2x1 (Is it one outcome or two?)

- Some students may list impossible products like 11 or 17

Some students may attempt to use a tree diagram to figure out the theoretical probabilities of odd and even products.
• Some students may simply figure out the products and not if they are odd or even

• Some may not leave enough room between the branches of the first die to put all six outcomes of the second die on each branch of the first die.

Some students may make simple computational errors

• Some students may miscount the number of odd products or total outcomes

• Some students will have computational errors like 2X3=5

Some students may try making a list or table of values exhausting all possible combinations of dice.

• Some students may get confused with the idea that there are two ways to have a 3 and a 1 come up on the dice. A 3 on the first die and a 1 on the second, or a 1 on the first die and a 3 on the second

• Some students, who “get” the concept above, may try to apply it to doubles like 1 and 1. This may cause them to list doubles in their table twice when they should only be listed once.

• Some may simply have duplicate or missing rolls simply due to a lack of organization of their list or table.

Some students may get so caught up in the list/table that they do not remember to keep track if their product is odd or even

Some students may try to list all possible outcomes in a table/area model/array with 1 through 6 on two sides and products in the table.

• Some may get caught up in the table and sums and forget to check if their product is odd or even.

Ask, “Are your products are odd or even?”

(Students may fix this error by stretching out branches, or separating branches into clusters)

Ask the students to check their computation if the teacher catches this error during the explore. If not, it should show up in the share.

Show the students a 3 red and a 1 blue and a 1 red and a 3 blue and ask them if these are two different outcomes (we never saw this).

Show the students a red 1 and a green 1 and ask them, “How would you change these dice to make the “other” 1-1 you have listed in your table?”

Ask the students to check with their group members to make sure they have all possible outcomes

Ask, “What outcomes are we interested in?”

Ask, “What outcomes are we interested in?”

Final Report of the Spring 2010 PMS Math Lesson Study Team  Page 18
Some students may try to determine the theoretical probability using an area model with odd and even on two sides and odd and even on the inside.

- Some who try this method may not know the product rules for odd and even products.

- Some who try this will not understand that odd x even is not the same outcome as even x odd (this leads to a theoretical probability of 1/3)

Some students may compare odds to evens instead of odds to total outcomes

Some pairs may simply report their experimental data as their theoretical data.

Some students may still insist that the number of possible outcomes is only two, odd and even, so the possibility of each must be 1 out of 2, or 50%

Some students will finish early

The teacher will be facilitating student learning by moving from pair to pair keeping students on task and asking guiding questions. The teacher should be watching for as many methods as possible even if those methods are simply different looking versions of the same method like a vertical and a horizontal tree diagram.

**Share:**

Have students share and explain their thinking and their methods using the document camera. Share two three or four methods (dependent upon time and number of methods attempted) in the following order.

| Ask, “Can you show me this with real numbers?” | Ask, “Show me examples of these with numbers?” If they are still having trouble, then show the students a 3 red and a 1 blue and a 1 red and a 3 blue and ask them if these are two different outcomes. |
| Have the students label what the numerator and denominator of their probability are and compare it to what it should be on page 5. | Have the students label what the numerator and denominator of their probability are and compare it to what it should be on page 5. |
| Ask “Tell me the difference between experimental and theoretical probability.” If the students make a reasonable response, ask them if they figured out the correct type. If they don’t know the difference, have them go over page 5. | Ask “Tell me the difference between experimental and theoretical probability.” If the students make a reasonable response, ask them if they figured out the correct type. If they don’t know the difference, have them go over page 5. |
| Ask, “Are we just rolling one die or are we rolling two?” next time around ask “Does this change your outcomes?” | Ask students to write down their thoughts about this question, “There is another way to deal with this without using 36, can you figure it out?” |
1. Tree Diagram
   - It is the easiest to see and understand
   - All students saw it the day before
2. Organized List
   - It is the next easiest
   - Relate it to the tree diagram
   - Discuss the two ways to roll a 3 and a 1
3. Table/Array with 1 through 6 on sides
   - It is the next easiest.
   - Discuss where 1,3 and 3,1 show up in the table/array
4. Area Model with odd and even on sides
   - It is the hardest to understand
   - It will probably not come up
   - Compare it to the Table with 1 through 6 to help students see where the odd and even pattern comes from

**Summarize:**

The teacher goes over the Probability Hypothesis Cycle and points out how we used it. The teacher then suggests that the class should now see how close their experimental results were to the theoretical results. Each pair figures out and reports their experimental data for the number of times Player A won while playing the game. The teacher records the data in a spreadsheet projected onto the board (see the blackboard plan).

The teacher leads a quick discussion about how “spread out” the data are. (Depending on time, this may be very teacher led) The teacher then suggests the class adds all the data together and see what happens. The teacher sums all player A wins and computes the percentage (this will done in an excel spreadsheet so continuous running percentage can be computed and possibly graphed very quickly) As the teacher adds more data, he/she continuously asks questions about what is happening to the experimental probability (individual and cumulative) and how it compares to the theoretical probability as more data is added.

Most students should realize that the experimental probability approached the theoretical probability as more data were added (the theory of large numbers).

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**Section 5:**

**Team Member 2010 Accomplishments**

Doug Lutz:

- Selected to attend the 2010 Park City Math Institute
Rey Munoz:
- Selected as a member of the LIFT grant at New Mexico State University
- Selected as a lead teacher for the 2010 MC² Summer Math Teacher Academies

Abram Sanchez:
- Selected as a lead teacher for the 2010 MC² Summer Math Teacher Academies

Tom Taney:
- Selected to present several break-out sessions during the 2010 MC² Academies
- Selected to help lead the New Mexico Video Table for the 2010 Park City Math Institute morning math sessions

MC² stands for Mathematically Connected Communities. It is the Math Science Partnership (MSP) Grant associated with New Mexico State University. MC² runs secondary math teacher academies for hundreds of New Mexico math teachers every summer.

Section 6: Record of Meetings, Topics, and Progress

1/25/10 First Team Meeting
Participants:
Susana Salamanca, Tom Taney, Rey Munoz, Abram Sanchez, Doug Lutz, and Linda Prendez

Objectives of the meeting:
- For members to meet each other (One team member comes from a different school, and Susana from NMSU attended the meeting)
- To discuss a broad overview of what lesson study is and what the process looks like
• To decide on a content strand for the lesson study goal based upon Picacho Middle School testing data (Number and Operations is our lowest content area) and curriculum

• The math PLC at PMS is studying the Process Standard of Reasoning and Proof

Accomplishments of the meeting:
We decided on the content standard of Data Analysis and Probability. We further decided to concentrate in the area of probability. The sixth grade teachers at PMS not normally get to the 6th grade probability book (How Likely Is It?) until after the NMSBA if at all. The 7th grade teachers do a two week mini-unit prior to the NMSBA. The 8th grade teachers don't even have a probability unit.

2/1/10 Second Team Meeting
Participants:
Tom Taney, Abram Sanchez, and Linda Prendez

Objectives of the meeting:
• Decide on goal and rationale

• Choose a lesson to teach with probability

Goals
• Use probability to generate arguments, draw conclusions and make decisions (Strand: Data Analysis and Probability; Standard: Students will understand how to formulate questions, analyze data, and determine probabilities; Benchmark: Understand and apply basic concepts of probability; 7th and 8th grade Performance Objective #5)

More specifically for this lesson:
• Students will use a probability model (tree diagram, area model, list of possibilities, etc.) to reason through and justify the fairness of a game

• Students will make connections between various probability models.

• The lesson will reinforce a Number and Operation concept (odd and even products) learned in the sixth grade unit Prime Time

Rationale
We decided on the content standard of Data Analysis and Probability. We further decided to concentrate in the area of probability. The sixth grade teachers at PMS do not normally
get to the 6th grade probability book (How Likely Is It?) until after the NMSBA if at all. The 7th grade teachers do a two week mini-unit prior to the NMSBA. The 8th grade teachers don’t even have a probability unit.

The team decided to use an existing lesson (Investigation 1, Problem 1.3) in the seventh grade book What Do You Expect?: Probability and Expected Value.

2/8/10 Third Team Meeting:
Participants:
Tom Taney, Abram Sanchez, Linda Prendez, Doug Lutz

Goals for this meeting were
• Keep working on the goal of the lesson.
• Choose the activity for the lesson based upon the more specific goal.

Accomplishments:
• We narrowed our primary goal to having students understand the theory of large numbers.
• We chose to use the dice product game activity in problem 1.3 of the CMP book What do You Expect

We are still working on the final language of the goal, and we are still having a lot of discussion about possibly changing the goal. Tom wants to include something about various representations of the theoretical probability in the goal. Doug still wants to bring in something about the process of coming up with a theoretical probability as part of the goal.

2/8/10 Doug and Tom meet with Susana:
Participants:
Tom Taney, Doug Lutz, Susana Salamanca

Goal for this meeting:
Susana wanted to meet and discuss the progress of the Picacho Lesson Study Team

Accomplishments:
- We came up with a representation of the process of coming up with a theoretical probability and decided to present it to the team at our next meeting. Doug and Tom want the team to include it in the goal of the lesson

2/22/10 Fourth Team Meeting:
Participants:
Tom Taney, Linda Prendez, Abram Sanchez, Rey Munoz

Goals:
- Finalize the goal(s) of the lesson
- Start talking about the flow of the lesson (essential questions, order in unit, etc.)

Accomplishments:
- We wrote two goals
- We started discussing the flow of the lesson

Our two Goals for the lesson are:
1. "Students will gain an understanding of the theory of large numbers" (with more and more trials, experimental probabilities approach theoretical probabilities)
2. "Students will improve their ability to use experimental probability data to verify or change their predictions about a theoretical probability"

2/29/10 Training with Blake and Observing Two Lessons:
Participants:
Tom Taney, Linda Prendez, Abram Sanchez, Rey Munoz, Doug Lutz

Goals:
- Participate in Training about Lesson Study
• Observe a Public Lesson
• Observe a Practice Lesson

Accomplishments:
• We attended Lesson Study Training with Blake and observed a public and practice lesson

2/30/10 Training with Blake and Lesson De-Briefs:
Participants: Tom Taney, Linda Prendez, Abram Sanchez, Rey Munoz, Doug Lutz

Goals:
• Participate in Training about Lesson Study
• De-Brief a Public Lesson
• De-Brief a Practice Lesson

Accomplishments:
• We attended Lesson Study Training with Blake and attended the de-briefing of yesterday's public and practice lessons.
• We shared our lesson ideas with Blake and came up with another possible goal for our lesson "Students will gain an understanding that the possible outcomes (the denominator of a theoretical probability computation) must be equally likely"
• We decided to teach the lesson over two days. The first day would be an introduction to the "Hypothesis-Experimental Probability-verify/change Hypothesis-Theoretical" Cycle, and the students would make an initial hypothesis/prediction about the fairness of the dice product game, and play the game 36 times. The second day would start off with students verifying or revising their predictions from the day before, determining a theoretical probability, and then looking the relationship between the theoretical probability and the experimental probability as more experimental trials are added.
3/8/10 Sixth Team Meeting
Participants: Tom Taney, Abram Sanchez

Goals:
- Write the lesson plan (not just the goals, but the flow of the lesson)

Accomplishments:
- We wrote the initial lesson plan
- It included all of the first column (Steps, Learning Activities, Teacher Questions, and Expected Student Reactions), but very little of the second column (Teacher Responses to Expected Student Reactions)
- Doug attached the plan to the wiki page the following day after I e-mailed it to him

3/10/10 Practice Lessons in Rey Munoz’ Classes
Participants: Rey Munoz (teacher), Linda Prendez, Susana Salamanca, Tom Taney, Abram Sanchez

Goals:
- Observe student thinking and learning during the lesson to improve the lesson

Accomplishments:
- Rey taught, and the others observed the lesson

3/15/10 Seventh Team Meeting
Participants: Rey Munoz, Tom Taney, Doug Lutz, Susana Salamanca, and Linda Prendez

Goals:
- Review student thinking during the practice lessons
• Improve the first column of the lesson plan (Steps, Learning Activities, Teacher Questions, and Expected Student Reactions)

• Complete the second column of the lesson plan (Teacher Responses to Expected Student Reactions)

**Accomplishments:**

• We made a lot of changes to the first column of the lesson plan

• We started adding teacher responses to the second column of the lesson plan

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**3/29/10 Eighth Team Meeting**

**Participants:**
Rey Munoz, Tom Taney, Abram Sanchez, and Linda Prendez

**Goals:**

• Continue improving the first column of the lesson plan (Steps, Learning Activities, Teacher Questions, and Expected Student Reactions)

• Complete the second column of the lesson plan (Teacher Responses to Expected Student Reactions)

**Accomplishments:**

• We completed changes to the first column of the lesson plan

• We completed the second column of the lesson plan

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**4/5/10 Practice lesson in Tom Taney's 3rd period class**

**Participants:**
Tom Taney (teacher), Linda Prendez, Rey Munoz, Abram Sanchez

**Goals:**

• Observe student thinking and learning during the lesson to improve the lesson

**Accomplishments:**
Tom taught, and the others observed the lesson

4/5/10 Ninth Team Meeting
Participants:
Tom Taney, Rey Munoz, Linda Prendez, and Doug Lutz

Goals:
• Review student thinking during the practice lesson
• Improve the lesson based upon observations of the practice lesson

Accomplishments:
• We made several changes to the lesson plan

4/19/10 Tenth Team Meeting
Participants:
Tom Taney, Linda Prendez, Rey Munoz

Goals:
• Improve the Summary Section of the lesson
• Write the excel spreadsheet for the summary

Accomplishments:
• Re-wrote parts of the summary section of the lesson
• Started working on the excel spreadsheet formulas for the summary

5/3/10 Eleventh Team Meeting
Participants:
Rey Munoz, Tom Taney, Abram Sanchez, Linda Prendez, Susana Salamanca

Goals:
• Go over the excel spreadsheet and chart for the summary of the lesson
• Assign administrative duties to team members for the public lesson on Friday

Accomplishments:
• We tweaked the use of spreadsheet in the summary
• We assigned team members administrative duties for the public lesson

5/7/10 Our Public Lesson and De-Brief
Participants:
Tom Taney (teacher), Rey Munoz, Abram Sanchez, Linda Prendez, Doug Lutz, members of the 3 other BLT lesson study teams, all Picacho math teachers, and Blake Peterson

Goals:
• Observe student thinking and learning during the lesson to check our hypothesis that the lesson will accomplish its goals

Accomplishments:
• Tom taught, and the others observed the lesson
• We heard the thoughts of other observers and of Blake Peterson during the de-brief
• We responded to the thoughts of others during the de-brief

5/3/10 Eleventh Team Meeting
Participants:
Rey Munoz, Tom Taney, Abram Sanchez, Linda Prendez,

Goals:
• Make final changes to the lesson plan based upon the public lesson and de-brief
• Reflect on the process and take notes to write the team reflections
• Assign team member the task of writing their individual reflections

Accomplishments:
• We made a few changes to the lesson plan (described in our team reflections)
• We reflected and took notes to write the team reflection part of our final report