
**FIFTH NEW MEXICO ANALYSIS
SEMINAR**

**Department of Mathematical Sciences
New Mexico State University**

Las Cruces

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ABSTRACTS

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FOURTH NEW MEXICO ANALYSIS SEMINAR
University of New Mexico, New Mexico
February 21-23, 2001

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Main speaker: **John Benedetto** (University of Maryland)

Lecture 1. **Wavelet theory and a fundamental application**

Lecture 2. **Multidimensional wavelet theory: MRA and non-MRA**

Lecture 3. **Fourier frames and weighted Fourier transform inequalities**

Abstract for the series. *In the first lecture we present elementary wavelet theory, including an historical background and the role of the uncertainty principle. The fundamental application is also elementary to explain, and is motivated by classical problems in spectral estimation. Its origins are in the problem of epileptic seizure prediction. The second lecture presents the theory of multidimensional wavelet theory, and provides basic constructions of single dyadic orthonormal multidimensional wavelets. These constructions were unexpected in the mid-1990s and are intimately related with fractal sets and the notion of self-similarity. The third lecture deals with the theory of Fourier frames, versus the wavelet frames of the first lecture. In the finite case, we study Platonic solids and in the infinite case we solve a data acquisition problem in MRI. We close by proving new Fourier transform norm inequalities, which, in turn, give rise to uncertainty principle inequalities of the type which arose in the first lecture. The speaker's own results in these talks represent joint work with David Colella, Matthew Fickus, Hans Heinig, Goetz Pfander, Songkiat Sumetkijakan, and Hui-Chuan Wu.*

INVITED TALKS:

Arpad Benyi (University of Kansas, Lawrence)

Bilinear singular operators, smooth atoms and molecules.

Abstract: *A class of bilinear pseudodifferential symbols is shown to produce bounded operators from $L^2 \times L^2 \rightarrow L^1$. The result is a bilinear substitute of a celebrated theorem of Calderon and Vaillancourt for linear pseudodifferential operators. The symbols $\sigma(x, \xi, \eta)$ are bounded functions with some decay in the frequency variables ξ and η , but they do not possess any smoothness in the x -variable.*

Marcin Bownik (University of Michigan)

Quasi-affine frames for rational dilations

Abstract: *In this talk we will show how to extend the notion of a quasi-affine system, originally introduced by Ron and Shen for integer expansive dilations, to the class of rational expansive dilations. One of the fundamental results in this area states that a usual wavelet system (=affine system) is a tight frame if and only if its quasi-affine counterpart is also a tight frame. We will present several interesting consequences of this result.*

Chris Brislawn (Los Alamos National Laboratory)

Group-theoretic factorization of linear phase wavelets.

Abstract: *We present a group-theoretic structure for cascade-form synthesis of linear phase filter banks and regular wavelets with odd-length impulse responses. The structure is amenable for factorization and efficient implementation of known filter banks as well as optimal design of new filter banks. We illustrate by showing the factorization of the well-known Cohen-Daubechies-Feauveau 9-7 filter bank and an optimal design of a 7-tap/5-tap filter bank with regular wavelets that will appear in Part 2 of the JPEG-2000 standard.*

Stephen D. Casey (American University)

Sampling on Unions of Incommensurate Lattices

Abstract: *Work on multichannel deconvolution has led to the development of new sampling sets. These sets are the unions of regular lattices, with each lattice generated by a rate incommensurate with all other rates. These new sampling schemes have in turn led to a new direction in A-D conversion. This talk will discuss these schemes and their application to A-D conversion.*

Peter G. Casazza (University of Missouri)

Applications of Hilbert space frames.

Abstract: *Hilbert space frames have traditionally been used in signal/image processing. Recently, there have arisen a variety of new applications to speeding up the internet, producing cell phones which won't fade etc. We will review the myriad of applications of frames to internet coding, multiple-antenna coding, quantum mechanics and Financial Mathematics with an emphasis on the open problems.*

Magali Folch-Gabayet (Instituto de Matematicas, UNAM Ciudad Universitaria)

L^2 theory for a class of oscillatory singular integrals.

Abstract: *We prove the convolution operator with kernel $K \exp(1/P)$, where K is a Calderon-Zygmund kernel and P is a real polynomial defined on R^n , with $P(0) = 0$, is bounded on L^2 , with bounds independent on the coefficients of P .*

John Gilbert (The University of Texas at Austin)

Frames and pointwise convergence..

Abstract: *When operators are represented as sums of wavepackets time-frequency methods can be applied. I shall use such 'time-frequency' frames in discussing the pointwise convergence of Fourier series.*

Martha Guzman-Partida (Universidad de Sonora, México)

S' -convolvability with the Poisson kernel.

Abstract: *We obtain real-variable and complex-variable formulas for the integral of an integrable distribution in the n -dimensional case. The integrands in these formulas involve specific versions of the Poisson kernel, namely, the euclidean version and the product domain version, and it is possible to reinterpret these integrands as S' -convolutions with such kernels. We also characterize the tempered distributions which are S' -convolvable with each of these two versions of the Poisson kernel. The distributions obtained on each case, turn out to be weighted versions of the space of integrable distributions.*

Maurice Hasson (The University of Arizona)

Littlewood-Paley theory and detection of singularities

Abstract: *We will show how wavelets with many vanishing moments, together with the classical Littlewood-Paley decomposition, allow us to detect changes in derivatives of a signal.*

Jeff Hogan (University of Arkansas, Fayetteville)

Sampling and oversampling in PSI spaces.

Abstract: *In this talk we describe new results in the sampling of signals in principal shift-invariant and multiresolution spaces. Criteria based solely on the length of the support of scaling functions will be given which ensure the validity of critical sampling and oversampling algorithms. Applications to the determination of the shift of oversampled data will also be given. This is joint work with Joe Lakey (NMSU).*

Mark Lammers (Western Washington University)

Gabor Frames and Hilbert C^* Modules.

Abstract: *We construct Hilbert C^* -modules useful for studying Gabor systems. We will discuss the role of the Zak transform and Bessel Systems in this setting and show that the preframe operator acts as a modular convolution for the Hilbert C^* -module.*

Manuel Leon (University of Missouri Columbia)

Lifting and FT on finite Abelian Groups

Abstract: *Decimation by a factor of m and subband coding in m -channels filter banks takes place on signals that, considered as abelian groups, contain a subgroup isomorphic to Z^m . In general, a finite d -dimensional signal is a finite abelian group. Every subgroup generates a quotient group. Elements of the quotient subgroup correspond to sublattices, or decimated versions, of the original signal. In this context subband coding and lifting is naturally described, and previous work by I. Daubechies and W. Sweldens on second generation wavelets is generalized to arbitrary n -dimensional signals.*

John N. Mc Donald (Arizona State University)

Phase retrieval for polynomials on the unit circle.

Abstract: *We discuss the following result: Let $P(z)$ and $Q(z)$ be polynomials satisfying: $|P(z)| = |Q(z)|$ and $|P'(z)| = |Q'(z)|$ for all z with $|z| = 1$. Then $P(z) = cQ(z)$ for some constant c .*

Osvaldo Mendez (University of Texas at El Paso)

The Poisson Problem with Robin Boundary Condition in non-smooth domains.

Abstract: *We Solve the Poisson Problem for the Laplacian, with Robin Boundary condition in a Lipschitz domain. We consider the right-hand side in the potential space L^p_{-t} , and use layer potentials to solve the problem in the class L^p_{2-t} .*

Tejinder S. Neelon (California State University San Marcos)

One-dimensional Restrictions of Smooth Functions on \mathbb{R}^n

Abstract: *It is a well known theorem of J. Siciak states that if $f \in C^\infty(\mathbb{R}^n)$ is such that its restriction to any line in \mathbb{R}^n is a real analytic function, then f necessarily is real analytic as a function of n variables. Various generalizations of this result are obtained by replacing the class of real analytic functions by function spaces of ultradifferentiable functions, the hypotheses on f are made only on a countable set of lines through each point, and finally, by replacing lines by $C^\infty(\mathbb{R}^n)$ curves. Even in the real analytic case, Siciak's theorem is improved.*

Kasso Okoudjou (Georgia Institute of Technology)

Embeddings of Some "Classical" Banach Spaces Into Modulation Spaces

Abstract: *We give sufficient conditions for a tempered distributions to belong to certain modulation spaces by showing embeddings of some non-homogeneous Bessov-Triebel-Lizorkin spaces into modulation spaces. As a consequence we have a new proof that the Hölder-Lipschitz space $C^s(\mathbb{R}^d)$ embeds into the modulation space $M^{\infty,1}(\mathbb{R}^d)$ when $s > d$. This embedding plays an important role in interpreting recent modulation space approaches to pseudodifferential operators.*

Manos Papadakis (University of Houston)

Generalized Frame Multiresolution Analysis and Frames of Translates

Abstract: *The Generalized frame multiresolution analysis (GFMRA) of abstract Hilbert spaces is an MRA structure that encompasses orthogonal, biorthogonal both one dimensional and multidimensional MRA structures. Their core subspace, usually denoted by V_0 , has a frame which is produced by the action of an abelian group of unitary operators (called Translation group) on a countable set of vectors called set of scaling vectors. We prove that there exists a countable set of wavelet vectors associated with this structure in the sense that the action of the Translation group on this set produces a frame for the orthogonal complement of the core subspace with respect to its immediate successor subspace V_1 . We also generalize the concept of frames of translates for abstract Hilbert spaces and characterize the sets of vectors generating such frames.*

Salvador Perez-Esteve (Instituto de Matemáticas, Unidad Cuernavaca, México)

Reproducing kernels for spaces of eigenvalues of the Dirac operator in the Plane.

Abstract: *We study spaces of solutions of the equation $DF = F$, where F is a function in the plane with values in the quaternions and D is the Dirac operator. The functions F will belong the Sobolev type space of all functions in $L^p(\Omega, |x|^{-3}dx)$ jointly with their angular and radial derivatives, and where Ω the complement of the unit disk in \mathbf{R}^2 . The resulting spaces are right Banach modules over the quaternions. When $p = 2$ we calculate the reproducing kernel of this space and explain its reproducing properties when $p \neq 2$.*

Cristian Rios (McMaster University)

Degenerate Monge Ampère equation in two dimensions.

Abstract: *Necessary an sufficient conditions are obtained for the existence and uniqueness of $C^{1,1}$ solutions to the Monge Ampère equation. Both the right hand side and the domain are allowed to be degenerate.*