Mathematics Appreciation

Josefina Alvarez
New Mexico State University, USA

Abstract

Mathematics Appreciation is a terminal course designed for non-science majors, to fulfill their general education requirement in mathematics at New Mexico State University. Using the development of this course as a theme, I present here my observations on how to motivate mathematically desinclined students, by placing mathematics in the context of non-mathematical experiences. I include numerous comments from my students. In closing, I argue that the appreciation of mathematics is in fact a concept to be emphasized in any mathematical course.

I. An overview Located in Las Cruces, New Mexico, in a mid-size city environment, New Mexico State University-Main Campus (NMSU) is a public (state) institution founded in 1888. About 42% of the NMSU students are Black, Hispanic, or Native American. NMSU satisfies the US Department of Education criteria for minority institution status and it is classified by the Carnegie Foundation for Education as a Research University I.

The course Mathematics Appreciation is a one semester course, mandatory for students not taking Calculus. The prerequisites are High School Algebra and an adequate score on a Mathematics Placement Examination.

The Department of Mathematical Sciences at NMSU offers each semester 10 to 12 sections with 40 students in each section. The student body is extremely diverse in gender, ethnicity, preparation, attitude towards mathematics, and area of study.

The course is part of the University General Education Program. The characteristics of the program include use of the library and a literature search, a substantial writing component, a historical context, an international perspective, and a multicultural influence.

- The specific goals of this course are to provide students with some understanding of the role of mathematics in civilization and with the ability to read, understand, and use mathematics.

The course does not have a prescribed syllabus. Each instructor is encouraged to pursue her or his own ideas of how to fulfill the goals of the course. Instructors have access to a large collection of classroom materials and guidelines contributed by past instructors and updated each semester. The course has a coordinator who provides guidance to new instructors, updates the materials, and supervises textbook selection for those instructors who choose to use one.

Since the Spring of 1994, part of my teaching duties have been to teach each semester a section of this course and to serve as course coordinator. What follows is a description of my approach to teaching this course.

- In the end, what I will be talking about is the fundamental role of motivation and illustration in the teaching of mathematics. This is the underlying concept that transcends any specific course.

II. What do I mean by mathematics appreciation? As H. O. Pollak (Pollak, 1996) very accurately observed, 'the perception most people have of mathematics has been molded by their educational experience, and neither the experience, nor its recollection tends to be happy.' Ironically, many people dislike and fear a mislabeled enemy, because they have only seen what David Fowler (Fowler, 1994) calls schoolmath, a quite different subject with its own terminology, methods, and beliefs. Although it is true that most nonscience majors may have forgotten a good part of their schoolmath, the bad feelings about their experience remain.

On the other hand, in spite of the undisputable applicability of mathematics, it is true that it takes some planning to communicate the effectiveness of mathematics to a nonspecialized audience.

As a result of all these factors, the dealings of mathematics seem too often closed off as by a high wall. How do we breach this wall, how do we present mathematics in a way that a passerby may enjoy it? Better yet, how do we lure a reluctant spectator into becoming, to some extent, a performer?

- I do not think that there is a universal recipe for what constitutes to appreciate mathematics.

However, it is my experience that any successful approach should recognize the special characteristics and the wealth of nonscientific knowledge that the students have. In the words of D. F. Halpern (Halpern, 1997), 'What and how much students learn in any situation depends heavily on their prior knowledge and experience...Because students frequently fail to apply what we have taught them in class to the real world, we must focus part of our teaching on "transferability". '

- I find very effective to present mathematics in several combined ways: As a powerful tool in the students' own business, as a mean to develop effective

thinking and communication skills, as a phenomenon of cultural history, and as a collection of fundamental thoughts and ideas.

I think that one of my tasks is to convincingly present the central role that mathematics has played in people's lives throughout history. Another task is to convey what the physicist Eugene Wigner called 'the unreasonable effectiveness of mathematics'.

This effectiveness was already observed in 1920 by Albert Einstein when he asked 'How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?'

III. How do I teach mathematics appreciation?

- I do not think that a course on mathematics appreciation should insist on technical skills.

Rather, it should make the best out of whatever skills the students have, typically a vague recollection of high school algebra.

- Still, I view this course as a mathematics course, where the students are expected to see and do mathematics. In fact, many of the most beautiful and enduring mathematical ideas and their applications, have a common sense quality that makes them fairly natural to grasp.

For instance, the irrationality of $\sqrt{2}$ or that there are infinitely many prime numbers. Or Erathostenes' method to find the circumference of the Earth:

(insert picture 1 here)

If Wiles' proof scapes this common sense quality, still the story of Fermat's last theorem is a thriller, nicely staged in the BBC video 'The Proof'[1], which shows that mathematical creativity is as passionate as any other human endeavour. How about counting? One can go a long way using the idea of counting the fingers in one's hand, one, two, three, four, five, or, in other words, establishing a bijection between the five fingers and the collection of five numbers $\{1, 2, 3, 4, 5\}$.

(insert picture 2 here)

When we allow our collections to have infinitely many members, some strange things happen. For instance, there are hotels where there is always a vacancy. Indeed, imagine a hotel with infinitely many rooms, numbered 1, 2, 3, 4, 5, 6, ..., .

(insert picture 3 here)

[1]NOVA, Show # 2414, October 28, 1997.

The hotel manager knows that all the rooms are occupied. However, when a new passenger arrives asking for a room, she says, "no problem". How come? Well, the manager moves the person in room 1 to room 2, the one in room 2 to room 3, and so on.

(insert picture 4)

Now room 1 is empty.

- I contend that practically every mathematical topic, appropriately organized, can be turned into an engaging presentation.

The instructional format that I use is highly interactive, with students doing a significant amount of the work in class, in groups. I require students to have and use a scientific calculator. The calculator is a good and simple way of fighting the tendency students bring to look for 'the answer', to learn about rounding-up or rounding-down, and to practise little numerical experiments.

I use readings from books, journal articles, newspaper and magazine articles. I also include in-class activities such as problem solving and discussion, visual materials, and hands-on materials.

- One major goal is to exemplify the presence of mathematics outside any 'mathematical environment'. This is what I call 'accidental mathematics'.

For instance, in a newspaper report on an archeological finding in Southern Chile, one can read 'Radiocarbon dating of bone and charcoal...' Another article on wine production states that 'Growers prefer small berries, because much of the flavor and color of grapes is in the skin. Large berries have a lower surface-to-volume ratio, and thus have proportionately less skin.' Yet another article reports on state appropriations for higher education. Apparently, our university has gotten a positive increase in budget, which, alas, turns out to be negative when corrected for inflation.

- I place great emphasis on the historical development of concepts, on the recognition of patterns, and on the sociological implications of mathematical information and choices.

Along the semester I ask students to write short opinion essays. They are quite surprised to find subjectivity and the possibility of interpretation in some aspects of a topic frequently thought of as an absolute truth. Besides serving as one of the parameters to measure their learning process, these essays provide me with an informal feedback on their changing views of mathematics. I quote here from their essays:

'Never in any geometry class have I learned who influenced the Greeks!'

'Societies function mathematically as well as is necessary for them.'

'If the human mind has simply manufactured the idea of mathematics, why does it work so well?'

'Mathematics was already there for man to discover and use.'

'The more I write, the more I think mathematics is a "human thing" not an everlasting subject.

'Mathematics is then as natural for humans as language and communication.'

Students also work on two significant writing assignments based on topics of their choice. As a guide, I provide the students with an extensive list of references.

I limit my lecturing to a minimum. My role in the classroom is more of a moderator of students' discussions. I am fortunate to use a classroom with movable furniture that encourages and facilitates cooperative work.

I start the semester asking the students to work on a set of very simple problems that I call 'Warm-Up Exercises'. The students are surprised to see that they can actually 'do some math' right from the start and without my intervention. I use these problems to prepare the ground from which the branches of the course will grow. For instance, the problem 'An employee's annual salary was increased from $\$21,582$ to $\$23,103$. What percentage rate did she get?, uses the same concept of rate that we will use later when we work with populations or investments.

Typically, topics I cover are mathematics in ancient civilizations, mathematical ideas of traditional peoples, numbers, modeling population dynamics, consumer mathematics, puzzles and paradoxes, and mathematics in the art of M. C. Escher.

- I prove and discuss some of the formulas to be used. Even very simple formulas, such as compounded interest or annuities, provide powerful examples of mathematics at work.

For most students, this is the first time they see how a formula is obtained. It also gives a meaning to algebraic manipulations they may have seen before.

- Overall, my purpose is to emphasize the continuity and connectedness in the way mathematics develops and progresses.

For this purpose, I find crucial to emphasize credible beginnings and links to previous experiences or human needs.

IV. Students Feedback I ask the students to answer the following questionnaires at the beginning and at the end of the course:

Questionnaire 1

I am seeking information on students' perceptions of this course. Your cooperation on filling out this questionnaire will be very useful. One word answers do not give me very much information. Feel free to add any comments. You do not need to write your name on this form. I will ask you to answer another brief questionnaire at the end of the course. I am asking you to write your date of birth to match your answers to both questionnaires. Thank you for your time.

- *What does mathematics appreciation mean to you?*
- *What do you expect to see in this course?*

Questionnaire 2

I am seeking information on students' perceptions of this course. Your cooperation on filling out this questionnaire will be very useful. One word answers do not give me very much information. Please, give reasons for your responses, and whenever appropriate give examples. Feel free to add any comments. Your responses will be held in the department's office, and provided to me only after the semester is over and grades are assigned. You do not need to write your name on this form. I am asking you to write your date of birth to match this questionnaire with the questionnaire you answered the first day of class. Thank you for your time.

- *How has this class been like or unlike your personal expectations? What parts of the class have been different from what you expected?*
- *Has your view of mathematics changed? How?*
- *Which topic was your favorite? Why?*
- *What topic was your least favorite? Why?*
- *Is there anything in this class that you would like to see changed?*

I give out Questionnaire 1 at the beginning of the first day of class. This questionnaire helps students reflect on their prior knowledge and attitude towards mathematics. It also gives me valuable information on the class profile.

Questionnaire 2 comes after the students have gone through the entire course. It helps them reflect on their learning process and on how this process has changed, or not, their perception of mathematics. It also helps me fine tune the course.

In their answers to Questionnaire 1 many students reject the possibility of appreciating mathematics. Their expectations on what they will see in the course are also vague. Many expect to see 'lots of algebra'.

The students' responses to Questionnaire 2 show, for the most part, a very positive change in their attitude towards mathematics. I quote here a few responses:

'I expected drudgery of trying to complete copious amounts of stupid problems such as if two pipes fill a pool that is unplugged...Instead I worked on things that I can go home and do for myself and my family, like retirement accounts.'

'This class has helped me understand that we need math in everything we do. I did not expect to look at art in a math class. I did not know math could be in art.'

'Math seems more like a philosophy now, a way of perceiving the world instead of something to use only to solve algebraic problems of no pertinent importance.'

'This is the first math class that I have actually enjoyed.'

'I found it refreshing to write essays on mathematical ideas with our opinion rather than just calculating problems.'

The official course evaluation administered by the department also shows a very high level of satisfaction.

V. Conclusion It could be argued that since this is a terminal course, students gaining an appreciation for mathematics will not have a chance to build on it in future courses. However, it should be observed that mathematicians, and scientists in general, are rarely found among political advisors and policy makers. So, if the educated public opinion has a better understanding of mathematics, this can have a positive influence in their response to the needs of the mathematical community. It can also produce an appreciation for the importance of mathematics in the school curriculum and beyond. Furthermore, an individual's perception of mathematics will undoubtedly influence their children's attitude towards the subject, thus helping to reinforce or brake the circle of mathematical avoidance.

- In a personal note, to teach mathematics appreciation has had a profound and positive effect in the way I teach any other course at the undergraduate or graduate level. Motivation, illustration, and historical development of the concepts are now high priorities when I am preparing to teach a course.

VI. References

- Fowler, D. (1994) *'What Society Means by Mathematics'*, Focus, Newsletter of the Math. Assoc. of Amer.,
- Halpern, D. F. (1997) *'The War of the Worlds: When Students' Conceptual Understanding Clashes With Their Professors'*, The Chronicle of Higher Education, 43, No. 27, March 14, 1997.
- Pollak, H. O. (1996) *'Mathematics: the Science of Patterns'*, Book Review, Notices of the Amer. Math. Soc., November 1996.

VII. Biography Dr. Josefina Alvarez was born in Spain and raised in Argentina. She has a Ph.D. in mathematics from the University of Buenos Aires, under Alberto P. Calderón, and she is currently a Professor of Mathematics at New Mexico State University.

Dr. Alvarez tries to maintain a difficult balance between her strong interests in harmonic analysis and in teaching and curriculum development.

In the area of teaching and curriculum development, she has published several papers and she has coauthored a manuscript titled 'Teaching Mathematics Courses Using Themes'. This manuscript has been submitted to the Mathematical Association of America. Dr. Alvarez has given several talks and workshops at professional meetings and universities, and she has conducted two NSF Short Courses for College Teachers, at the University of Texas at Austin and at the University of Puerto Rico, founded by the National Science Foundation. She is the 1998 recipient of the El Paso Energy Foundation Faculty Achievement Award, for "outstanding University teaching."